2 copie, spaziatura doppia, margine di almeno 3 cm

Brightness relations between retinal regions as necessary conditions

Achromatic colour conditions in the perception of transparency

Lavoro per il volume in onore di Gibson (prof. MacLeod)

Figure 3 e 4

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# ~~A RESEARCH ON THE PERCEPTION OF TRANSPARENCY~~

In the research on Perception, important results have been reached through the study of "illusions", namely phenomena where perception fails to give correct information about the objective situation.

In the case of the perception of transparency, illusions help us to "see" the very problem. In fact, under normal conditions, where we are perceiving as transparent what is permeable by luminous radii, there seems not to be any perceptual problem. But if we put a sheet of transparent plastic, or a piece of glass on an homogeneous background, we do not perceive it as transparent, while in situations like Fig. 1, where there is no physical transparency, we perceive transparency (1); hence it is natural to ask oneself the reason for these facts.

Fig. 1

(1) It could be objected that in this case, as in all the examples given in this paper the impression of transparency is incomplete, poor, and however different from a "true" transparency, as perceived when we look through a (coloured) glass of a window, or through the surface of the sea.
The reason for this difference is that in our pictures three-dimensionality is lacking, which is not a necessary, but a very powerful condition supporting the impression of transparency. But comparing real (= physical) transparency with apparent transparency under the same conditions of bidimensionality (that is, comparing a chess board with a film of grey transparent plastic on it, with a formally and chromatically identical model, but constructed as a mosaic by juxtaposition of 4 different shades of grey opaque paper, like Fig. 4) the impression is the same, and nobody can say where there is physical transparency and where not. 1

An initial explanation can be attempted in terms of cues: normal cues of transparency are lacking in the first case and present in the second (a surface seen through a film appears altered in its colour, because of absorbtion and reflection by the transparent surface). But with irregular figures, like Fig. 2 and 3, it is by no means clear why we perceive in Fig. 2 two surfaces one seen through the other, and not several juxtaposed blots as in Fig. 3.

Fig. ~~2~~1

Fig. ~~3~~2

In general terms, the problem of perceptual transparency can be stated as follows: under which conditions the stimulation of a group of visual receptors ~~of the retina~~ gives rise to the perception of two surfaces, one of which is seen through the other?

This problem has been evidenced and discussed by Helmholtz and Hering ~~(1)~~, and further subjected to experimental researches by Fuchs, Tudor-Hart, Heider, Koffka, Metzger, Kanizsa, Petter, Metelli.

It is easy to show that there are two orders of conditions of perceptual transparency, figural (1) and colour-conditions: in fact it is possible to abolish ~~perceptual~~ transparency by altering form (Fig. ~~2,3~~ 1,2) or colour (Fig. ~~4,5~~ 3 4).

The perception of

Fig. ~~4~~3

Fig. ~~5~~4

(1) H. Helmholtz - Physiologische Optik, p.407 and following.
E. Hering - Über die Theorie des simultanen Kontrastes von Helmholtz, 4. Mitteilung, Pflügers Archiv 43, 1888

(1) My previous paper (8) on this subject is chiefly devoted to a first analysis of the figural conditions of transparency

The subject of this paper is a research into the colour conditions in perceptual transparency. As three numbers are needed for the definition of a colour, while an achromatic colour is completely defined by one number, the index of reflectance or albedo (the light reflected by an unit area divided by the light it receives), it appeared affared suitable to begin by confining this study, to the field of achromatic colours, which from now on will be named simply colours.

The subject of this paper is a research into the colour conditions in perceptual transparency (1). As the purpose of this study was to find a quantitative law about the influence of colour on the perception of transparency, the first task has been to chose a quantitative expression of colour.

It is well known that for the definition of a colour not less than 3 numbers are needed, while for the definition of the different shades of grey, from white to black (which are named achromatic colours) only one number is needed, namely the index of reflectance or albedo, the amount of light reflected from a unit area divided by the amount of light it receives (the formula being $L = \frac{i}{I}$ , where $L$ stands for the coefficient of reflection or albedo, $i$ stands for the intensity of reflected light and $I$ for the intensity of light falling into the area). Since an absolute white reflects the whole amount of light that it receives, while the absolute black absorbs the whole light falling on it, the absolute white has reflectance 1, the absolute black reflectance 0 and the various shades of grey have coefficients of reflectance between 0 and 1.

For this reason - the achromatic colours being univocally defined by one number it appeared suitable to begin by confining my study to the field of acromatic colours, which from now on will be named simply colours.

Let us start from a simple figure (Fig. 5) where subjects normally perceive transparency. The figure has been chosen because it reproduces the situation of transparency obtained by an episcotister - a rotating wheel with open sectors -; the following considerations stand for both situations, but they are easier to follow if we start with a model where perceptual transparency is obtained by the method of juxtaposition of opaque surfaces (2).

Fig. 5 Fig. 6a

(1) This problem has been considered from a special point of view in F. Metelli - Zur Analyse der phänomenalen Durchsichtigkeitserscheinungen (Gestalt und Wirklichkeit, Festgabe für F. Weinhandl, Berlin, 1967), that is chiefly devoted to the theory and to a first analysis of the figural conditions of transparency.

(2) This method is due to W. Metzger (Gesetze des Sehens, II^ ed. 1953, pp. 127-8). ~~In the figures~~ of this paper ~~phenomenal transparency is obtained by juxtaposition of opaque surfaces.~~ The original figures, whose photographic reproductions are printed, are mosaics of cardboard (except Fig. 1 and 2 which are pictures)

4)

(Fig. 6) 

In this figure - which can be considered as a general model for perceptual transparency phenomena - we distinguish 4 different regions, with four different shades of grey; we name them A P Q B (capital) and the respective reflectances a p q b (small). The stimulation originated by the P region produces 2 different perceptual effects: we see an anterior layer T, which is transparent, and through this a second layer, the latter being of the same colour as the contiguous region A. (The same observation can be made for the region Q, but for the moment, let us confine our argument to P) (1). Therefore the perceptual phenomenon of transparency has been described as a case of perceptual scission - one of the much studied scission-phenomena where one sort of stimulation produces two effects, as for example surface-colour and illumination (2).

At this point it is natural to ask oneself what is the relation between stimulus colour and scission colours. In our figure we can easily perceive them all: the stimulus colour if we isolate the P region, and the scission colours when we perceive transparency.

A simple solution to the above problem is due to G.Heider and K. Koffka, who stated, and gave experimental proof that scission colours are such that, mixed together according to Talbot's law, reproduce the stimulus colour.

The example given by Koffka in his treatise (3) is as follows. If the stimulus colour is grey, and the conditions require that one of the scission colours be blue, then the other scission colour must be yellow. Symbolically, if Y + B = G, then G - B = Y.

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(1) It has to be stressed that every figure in this paper can be described according to the preceding scheme: there are always two non-splitting regions, A and B, which constitute the ground, and two splitting regions, P and Q, P splitting into A and T, and Q splitting into B and T. One needs only to add that most figures are serial repetitions of the sequence APQB alternated with the inverse sequence BQPA. (See for ex. Fig. 1,2,4 where two parts can be distinguished, an upper and a lower part, where the two inverse sequences are easy to identify; and Fig. 8-15, where several horizontal sequences are identifiable).

(2) In fact, in this case also, as in the colour constancy phenomenon, if we isolate the region which is represented twice (that is, in Fig. 6, the circle, which is perceived as a transparent layer and as a part of the ground seen through it) the perceptual scission disappears, and we perceive only one layer.

(3) K. Koffka - Principles of Gestalt Psychology, New York, 1935, pp. 260-264.

Heider and Koffka's theory is the starting point of this research.

It is clear that Heider and Koffka's formulation is not an algebraic formulation of the problem, because B , Y and G do not symbolize numbers. But it is possible to give an algebraic formulation to the problem if we confine ourselves to acromatic colours, using reflectance as their measure.

My line of reasoning has been the following. If according to Heider and Koffka's statement, the same law (that is Talbot's law in the special case of mixture through the colour wheel ~~(1)~~) rules colour-mixture and colour scission, then it is possible to use the law of colour mixture for an algebraic description of colour-scission.

Talbot's law says that if two acromatic colours, whose reflectances are a and b are mixed in equal quantities, the reflectance of the mixture c is the aritmetic mean of the two colours, $c = \frac{a+b}{2}$ . If the colours a and b are mixed in quantities m and n (m being the quantity of a and n the quantity of b) the reflectance of the mixture c is the weighted average of the components

$$c = \frac{ma + nb}{m+n}$$

or, in other words, the reflectance of the mixture-colour is the weighted average of the components, the weights being the quantities of the components.

The same formula can be expressed in a more suitable form if instead of the absolute quantities, we use as weights the proportions (summing up to 1) of the components. Putting $\frac{m}{m+n} = \alpha$ and $\frac{n}{m+n} = \cancel{1 - \frac{m}{m+n}} = 1 - \alpha$ the above equation takes the form (As $\frac{m}{m+n} + \frac{n}{m+n} = 1$)

$$c = \alpha a + (1 - \alpha)b$$

more suitable

$\alpha$ and $1 - \alpha$ being the proportions in which the components are present in the mixture.

But if Talbot's law rules both phenomena, colour mixture and colour scission, then the above equation, describing colour mixture, describes also colour scission. Since it is convenient to use symbols according to the preceeding example, the colour scission equation will be used from now on in the form

$$p = \alpha a + (1 - \alpha)t$$

(Fig. ~~5~~6)

where p is the reflectance of the stimulus-colour, a the reflectance of the second layer, t the reflectance of the first and transparent layer, and $\alpha$ and $(1-\alpha)$ are the quantities or more exactly

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(1) According to W. Ostwald ( Mathetische Farbenlehre, Leipzig 1921, p. 131) the law of colour mixture goes back to Newton.

the proportions into which the stimulus colour has been divided in the two layers (1).

2 But what meaning is to be given to quantities in this case? Or rather how will perceptually appear different quantities of colour when distributed to equal surfaces?

Different quantities distributed to equal surfaces can give as a result only a difference in colour density. And on the first layer a difference in colour density can appear only as a difference in transparency, while on the second layer it can reveal itself only through a difference in intensity which is correlative to the transparency of the first layer. In other words, great density on the first layer and little density on the second means little transparency of the first and little intensity or little visibility of the second layer; while less colour on the first layer means little density or great transparency, and more colour on the second layer means great intensity, and great visibility. That is

Let us check this interpretation with the equation. What happens if $\alpha = 0$, that is, if the first layer T gets the whole colour of P ? In this case the equation reduces to p = t, and the second layer, having no colour at all, is not visible: we see only the first layer T, which is completely opaque. Thus $\alpha = 0$ defines the limiting case of the perception of an opaque surface on the top of another surface, namely the usual figure-ground organisation (Fig. 7

And what happens if $\alpha = 1$? In this case it is the second layer A which gets the whole colour (the solution of the equation being p = a) and the first layer T, having no colour, disappears completely. This means that the first layer is perfectly transparent and therefore wholly invisible. (Fig 8) ← Sparisce la fig

In both these cases ($\alpha = 0$ and $\alpha = 1$) there is no colour scission and obviously no perception of transparency: colour scission takes place only in the inermediate cases, that is when $\alpha$ is between 0 and 1.

[Which then is the measing of the coefficient $\alpha$ , which has a maximum value 1 when the transparency is perfect, a minimum value 0 when there is no transparency at all, and has a high value when a little quantity of the scission-colour comes to the first layer, and therefore the transparency is great, and a low value if a great quantity of the scission colour comes to the first layer, and therefore the transparency is little?

The obvious inference seems to be that $\alpha$ - which measures the proportion of colour going to the second layer seen through the transparent one (the intensity or clearness with which the se

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(1) It is perhaps opportune to remember that the P region is divided into two layers, A and T, and the surface of each one is equal to the surface of P. (v. Fig. 5)

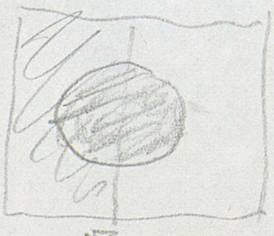

Fig. 7

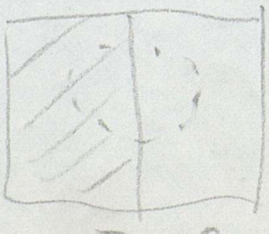

Fig. 8

Therefore $\alpha$ is a coeficient of transparency. It has however to be stressed that transparency depends also on another condition: the colour of the transparent surface. All other conditions being egual the fenomenal transparency of a surface is an inverse function of its reflectance.

cond layer is perceived) - measures the degree of transparency and is therefore a coefficient of transparency. But quantitative experiments show beyond any doubt that transparency depends also on the chromatic quality of the first layer, and therefore $\alpha$ is only a factor of transparency. A second factor is phenomenal colour, that is colour measured by the logarithm of the reflectance (1).

Thus $\alpha$ is the index of phenomenal colour-scission; it acquires the function of a coefficient of transparency only if the other conditions (first of all a and t colours (2)) are held constant. And, phenomenal colour scission being a necessary condition of transparency, the latter is possible only if $\alpha$ is less then 1 and more then 0.]

With this definition of $\alpha$ , the above equation should not contain any unknown symbol, as a and t are the reflectance coefficients which measure the colours of the scission layers. There is, however an important difference between a and t: a is the colour of the contiguous region A, and is therefore a known quantity (one of the known terms of the problem), while t, together with $\alpha$ , are the unknowns of the equation (3).

(Obviously) ~~In fact the problem of phenomenal colour scission can be given the following formulation:~~ starting from the knowledge of the

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[(1) All other conditions being equal, the transparency of an achromatic surface is an inverse function of the whiteness (or the reflectance) of the surface. The inverse relation between reflectance and phenomenal transparency can be easily shown using episcotisters of equal opening and different shades of grey.

(2) Experimenting with an episcotister it is easy to show that if the colour a (b) of the ground and the colour t of the rotating sector are kept constant, the transparency is a direct function of $\alpha$ , the open sector measured as a proportion of 360° ($\alpha = \frac{360 - \varphi}{360}$ , $\varphi$ being the size of the rotating sector).

(3) This sentence refers to the transparency cases experienced in everyday life or to the models discussed here like Fig. 1, 2, 4, 6 etc. In these cases we can measure the reflectances of the regions A and P (B and Q), while we do not know $\alpha$ and the reflectance of the transparent layer T. In the special case of transparency obtained through an episcotister the known terms of the problem are the colour of the ground a (b), the colour t of the rotating sector, and the size of the open sector $\alpha$.]

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colours (or reflectances) of the regions A and P, ~~is~~ it ~~possible~~ is not possible to predict the proportion $\alpha$ in which the colour p is divided among the two layers, and the colour t of the transparent layer, ~~t?~~

~~The answer is of course a negative~~ one, bacause the unknowns are two, and therefore the equation is indeterminate.

> ~~It is perhaps suitable to look more deeply into the question, because it can seem that, when the quantity p is divided into a and t, if p and a are known, t is determined.~~
>
> ~~A way of clarifying the question is to begin by putting it into very simple terms. If p is divided into equal parts, the equation simplifies to $p = \frac{a+t}{2}$ or $p = \frac{1}{2}a + \frac{1}{2}t$, and in this case once a is chosen, t is determined. If p is a point on a segment which represents the sequence of numbers from 0 to 1, which identifies the reflectances, that is the measures of achromatic colours, a and t have to be on opposite sides of and at the same distance from p. That is, p splits into two colours, each of which is half the quantity of p, and which, mixed together once again give p (the same colour in the same quantity).~~
>
> ~~But following the above line of reasoning we have renounced to one dimension of change, by fixing the proportion of splitting as 0,5 to 0,5. If we choose another splitting ratio, e.g. 0,25 to 0,75, and again consider p as a point on the 0-1 segment, a and t have also to be on opposite sides, but not at the same distance from p: the distance of t from p has to be 1/3 the distance of a from p. Is means that if p and a are fixed, the colour of t (that is, his position on the 0-1 scale of the reflectances) depends on the splitting ratio of p.~~

But so far we have used only one half of the A P Q B model (Fig. ~~5~~6) and thus only one half of the data. In fact we can write a second equation using b and q, that is

$$q = \alpha' b + (1 - \alpha')t'$$

and if we may put $\alpha = \alpha'$ and $t = t'$, which seems to be very often, if not always, right (1), the system of two equations with two unknowns is soluble. The solutions are

$$\alpha = \frac{p-q}{a-b} \qquad\qquad t = \frac{aq - bp}{(a+q) - (b+p)}$$

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(1) There are cases where the perceived transparency of P and Q is not the same, ~~As the colour of the transparent layer T appears the same on both regions (that is $t = t'$) it seems reasonable to hypothesize $\alpha \neq \alpha'$, namely that in these cases the quantity of colour assigned to T in the splitting process is different for the part of T covering A from that covering B.~~ and cases where only one of the P and Q regions is transparent, while the other is opaque. ~~In all~~ Obviously in all these cases above the solutions for $\alpha$ and t are not valid, and other couples of equations have to be written.

3 At this point it seems opportune to check the deduced formulas, that is to see if and to what extent there is a correspondence between theoretically deduced formulas and the facts.

Let us begin with the equation of the phenomenal scission index $\alpha$. The equation defines the field of transparency, because transparency is possible only for the values of $\alpha$ between 0 and 1; because as we have already seen $\alpha = 0$, (perfect opacity) and $\alpha = 1$ (perfect transparency) are the limiting cases where phenomenal scission is lacking, and therefore not to be considered among transparency phenomena. $\alpha > 1$ and $\alpha < 0$ would mean that one or the other layer would receive a negative quantity of colour, a situation which is devoid of meaning.

Therefore two necessary conditions follow from the formula

1. $|a-b| > |p-q|$ (otherwise $\alpha > 1$)
2. $(a > b) \Longleftrightarrow (p > q)$, $(a < b) \Longleftrightarrow (p < q)$ (otherwise $\alpha < 0$) 8 (pagina seguente)

The first condition says that the difference between the reflectances of the regions A and B has to be greater than the difference between the reflectances of the splitting regions P and Q (1). Negatively expressed it becomes a sufficient condition: if the difference in reflectance between the splitting regions P and Q is greater than the difference between the non splitting regions A and B, there cannot be transparency.

Fig. 9 Fig. 10 Fig. ~~9~~ 11

This condition can be easily controlled. Fig. 5 and 9 show two identical models with the only difference that in Fig. 5 the difference in reflectance between the internal semicircular regions is clearly less than the difference in reflectance between the external regions, while in Fig. 9 the contrary in true. The result is that in Fig. 5, and not in Fig. 9 transparency is perceived. It is interesting to see what happens in another sample of fi

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pagina successiva (1) As measures of reflectances are higher for lighter than for darker shades of grey, in terms of (achromatic) colour, the > symbol means "brighter than", and the < symbol "darker than". 0

drop size.

11

gures (Fig. ~~8~~ 10 and ~~9~~ 11) which are constructed following the same principle (1). In this case both figures can be perceived as transparent, but different regions are transparent in each figure; that is, the contiguous regions where the splitting phenomenon takes place are always the regions between which there is less difference in reflectance. In this case the relation $|a-b| > |p-q|$ has decided which regions take the functions of p and q, (that is, become the splitting regions) and which take the functions of a and b. Therefore in this case also, the above necessary condition has been respected (2).

The second condition can be espressed by saying that the brightness gradient (or the fall of brightness-level) between p and q on the one hand and between a and b on the other hand must have the same direction (3).

This condition also may be checked. On Fig. ~~4~~ 3, where transparency is generally perceived, this condition is respected because the splitting region which is contiguous to the brightest region A, the P region,is brighter than the other splitting region, Q. If we reverse the brightness gradient between P and Q (Fig. ~~5~~ 4) transparency is no longer perceived.

Fig. ~~10~~ 12 Fig. ~~11~~ 13

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(1) Fig. 18-~~17~~ are alternations of the sequences B A P Q B A and A B Q P A B. The critical sequences, necessary for the arousal of perceptual transparency are, of course A P Q B and (or) B Q P A. The addition of the squares A and B at the beginning and at the end of the sequence has only the effect of rendering coercive and therefore general among subjects the perceptual organisation which gives rise to phenomenal transparency. stabilizing

(2) This is, of course, only an initial and very rough check. ~~Other checks can be done constructing models where~~ $|p-q|$ approaches $|a-b|$~~. In these cases~~ the perception becomes ambiguous: there is alternation between transparency of the central region (like Fig. 18) and transparency of the peripheral region (like Fig. 19). In other words the law $|a-b| > |p-q|$ holds only when the difference between $|a-b|$ and $|p-q|$ reaches a certain size. 7f

(3) The formulation can be simplified to $(p > q) \Longleftrightarrow (a > b)$ or, yet further to $p > q$ if we define $a > b$, or, in other words, the brighter of the two shades of grey which form the ground, is named a.

(primo capoverso nella nota 3

Besides other, less interesting necessary conditions, it can be derived from the ~~splitting-index~~ above equation that if the colour t of the transparent layer T, is held constant (1), when the difference between a and b is much greater than the difference between p and q, there is little transparency (Fig. ~~10~~ 12), whereas when the difference between a and b is hardly greater than the difference between p and q, transparency is great (Fig. ~~11~~ 13).

4. The other formula, $t = \dfrac{aq - bp}{(a+q) - (b+p)}$ – which has also to be taken into consideration to define the field of transparency – is more complicated and does not offer the opportunity of deriving simple predictions. Nevertheless a way has been found to ~~derive~~ infer qualitative predictions also about the colour t of the transparent layer.

The original equation $p = \alpha a + (1-\alpha)t$ can be given the form $\alpha = \dfrac{p-t}{a-t}$. As it has already been said, in the case of transparency we have $0 < \alpha < 1$.

Let us consider first the disequation $\alpha > 0$, and therefore

$$1. \quad \frac{p-t}{a-t} > 0$$

This condition implies that numerator and denominator of the fraction are either both positive or both negative. We have therefore to take into consideration two cases.

CASE 1A

As numerator and denominator are positive, we have

$(p-t) > 0, \quad (a-t) > 0$

therefore [ $p > t$ and $a > t$

or

if $p > t$ then $a > t$ and vice versa

or ]

$$\boxed{(p > t) \Longleftrightarrow (a > t)}$$

$(p > t)$ implies and is implied by $(a > t)$

CASE 1B

As numerator and denominator are negative, we have

$(p-t) < 0, \quad (a-t) < 0$

therefore [ $t > p$ and $t > a$

or

if $t > p$ then $t > a$ and viceversa

or ]

$$\boxed{(t > p) \Longleftrightarrow (t > a)}$$

$(t > p)$ implies and is implied by $(t > a)$

Now let us consider the second disequation, namely $\alpha < 1$ and therefore

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(1) The necessity of keeping constant the colour of the transparent layer T depends on the formerly stressed fact that colour itself is a factor of transparency.

$$2. \quad \frac{p-t}{a-t} < 1$$

with reference to both cases A and B.

CASE 2A

As (a-t) is positive, multiplying both members of the disequation by (a-t), the direction of the disequation remains unchanged

$$\frac{p-t}{\cancel{a-t}}\cancel{(a-t)} < 1\ (a-t)$$

that is $(p-t) < (a-t)$

and therefore $\boxed{p < a}$

CASE 2B

As (a-t) is negative, multiplying both members of the disequation by (a-t), the direction of the disequation changes

$$\frac{p-t}{\cancel{a-t}}\cancel{(a-t)} > 1\ (a-t)$$

that is $(p-t) > (a-t)$

and therefore $\boxed{p > a}$

ASSOCIATING 1A and 2A

namely from the hypothesis that numerator and denominator are positive, that is

from $(p > t) \Longleftrightarrow (a > t)$

and $p < a$

it follows that

$$\boxed{a > p > t}$$

ASSOCIATING 1B and 2B

that is, from the hypothesis that numerator and denominator are negative, that is

from $(t > p) \Longleftrightarrow (t > a)$

and $p > a$

it follows that

$$\boxed{t > p > a}$$

Till now the consequences which followed from the above formula prove only what was a natural expectation, namely that when a phenomenal scission occurs, if one of the scission colours <u>a</u> or <u>t</u> is brighter than the stimulus colour <u>p</u>, then the other scission colour (<u>t</u> or <u>a</u>) is necessarily darker.

But so far we obtained the above inferences only about the areas A and P. Obviously, following the same line of thought, the same algebraic relations are obtained about the areas B and Q.

The conditions are, therefore, for the areas A and P

A. $a > p > t$ or B. $t > p > a$

and for the areas B and Q

C. $b > q > t$ or D. $t > q > b$

14)

Putting together each of the alternative conditions for the areas A and P with each of the alternative conditions for the areas Q and B, the following combinations are obtained.

| AC | AD | BC | BD |
|---|---|---|---|
| $a > p > t$ | $a > p > t$ | $t > p > a$ | $t > p > a$ |
| $b > q > t$ | $t > q > b$ | $b > q > t$ | $t > q > b$ |

The meaning of this operation is that by choosing a given brightness order for the regions A P Q B, we can predict the place of $\underline{t}$, the colour of the transparent layer, on the above sequences or brightness scales.

Of course, the occurrence of phenomenal transparency is not guaranteed by these combinations, because transparency conditions were taken into account for every half figure separately and not for the figure as a whole. Therefore, if the previously inferred necessary chromatic conditions of transparency are lacking, transparency cannot be perceived. The meaning of the combined brightness sequences is, therefore, that <u>if transparency occurs, then, from the brightness relations between the areas, the degree of brightness of the transparent layer can be predicted</u>.

From the preceeding analysis three different cases follow

1. The transparent layer T is the darkest of the sequence (case AC). If $\underline{a} > \underline{p}$ and $\underline{b} > \underline{q}$, $\underline{t}$ is the darkest of all. Combining conditions A and C, two sequences, $\underline{a} > \underline{p} > \underline{b} > \underline{q} > \underline{t}$ (Fig. ~~12~~) 14 and $\underline{a} > \underline{b} > \underline{p} > \underline{q} > \underline{t}$ (Fig. ~~13~~ 15) are generated which do not conflict with the necessary conditions of transparency (1).

Fig. ~~12~~ 14

Fig. ~~13~~ 15

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(1) $\underline{a} > \underline{p}$ and $b > q$ can also be combined to a sequence $a > b > \underline{q} > \underline{p}$, which would conflict with the necessary condition $(\underline{a} > \underline{b}) \Longleftrightarrow (\underline{p} > \underline{q})$ and in fact in this case we do not perceive transparency. The combination $a > (p=b) > q > t$ (where there are only 3 different shades of grey) does not conflict with the necessary conditions of transparency: in fact under these conditions transparency can be perceived.

2. The transparenct layer T is darker than a and p and brighter than q and b (cases AD and BC (1)). The sequence obtained by combining conditions A and D is $a > p > t > q > b$ (Fig. ~~8~~,10,~~11~~ 12, 13) which does not conflict with the necessary conditions of transparency.

3. The transparent layer T is the brightest of the sequence (case BD). If $p > a$ and $q > b$, t is the brightest of all. Combining conditions B and D, two sequences, $t > p > a > q > b$ (Fig. ~~14~~ 16) and $t > p > q > a > b$ (Fig. ~~15~~ 17) are generated, which do not conflict with the necessary conditions of transparency.

Fig. ~~14~~ 16

Fig. ~~15~~ 17

Fig. ~~12~~ 14, ~~13~~ 15, ~~8~~ 10, ~~14~~ 16, ~~15~~ 17 that correspond to cases 1a, 1b, 2, 3a, 3b can be considered an initial check of the above deductions. X

The theory can be applied also in special cases, when the different shades of grey are 3 instead of 4 (if a=p or a=q; a=b and p=q being excluded because in this case $\alpha$ is outside the field of validity); and also to the cases of more then 4 different shades of grey.

(1) Case BC repeats case AD if a instead of being the brightest, is the darkest of the four regions.

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Se $\alpha' = c\alpha$

$$p = \alpha a + (1-\alpha)t \qquad q = c\alpha b + (1 - c\alpha)t$$

$$p = \alpha a + t - \alpha t \qquad q = c\alpha b + t - tc\alpha$$

$$p - t = \alpha(a - t) \qquad q - t = c\alpha(b - t)$$

$$\alpha = \frac{p-t}{a-t} \qquad c\alpha = \frac{q-t}{b-t}$$

$$\alpha = \frac{q-t}{b-t}\,\frac{1}{c}$$

$$\alpha = \frac{q-t}{c(b-t)}$$

$$\frac{p-t}{a-t} = \frac{q-t}{cb-ct}$$

$$aq - at - tq + t^2 = pcb - pct - tcb + ct^2$$

$$t^2 + tcb + pct - ct^2 = pcb - aq + at + tq$$

$$t^2 - ct^2 + tcb + tcp =$$

$$t^2(1-c) + t(cb + cp) =$$

$$(1-c)t^2 + (cb - cp)t - pcb + aq - at - tq = 0$$

$$\boxed{x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$t_{1,2} = \frac{cp - cb \pm \sqrt{(cb-cp)^2 - 4(1-c)(aq - pcb - at - tq)}}{2-2c}$$

$$x_1 x_2 = \frac{c}{a}$$

$$x_1 + x_2 = -\frac{b}{a}$$

$$(1-c)t^2 + (cb + cp - a - q)t + aq - pcb = 0$$

$$p = \alpha a + (1-\alpha)t \qquad q = c\alpha b + (1 - c\alpha)t$$

$$\frac{p - \alpha a}{1-\alpha} = t \qquad \frac{q - c\alpha b}{1 - c\alpha} = t$$

$$\frac{p - \alpha a}{1-\alpha} = \frac{q - c\alpha b}{1 - c\alpha}$$

$$p - \alpha a - c\alpha p + ac\alpha^2 = q - c\alpha b - \alpha q + c\alpha^2 b$$

$$\alpha q + c\alpha b - \alpha a - c\alpha p + ac\alpha^2 - cb\alpha^2 = p - q$$

$$(ac - cb)\alpha^2 + (q - a + cb - cp)\alpha + q - p = 0$$

$$\alpha_{1,2} = \frac{a + cp - q - cb \pm \sqrt{(q - a + cb - cp)^2 - 4(ac - cb)(q-p)}}{2ac - 2cb}$$

$$ax^2+bx+c=0 \qquad p=\frac{b}{a} \quad q=\frac{c}{a}$$

$$x^2+px+q=0$$

$$x^2+px=-q$$

$$x^2+2\frac{p}{2}x+\frac{p^2}{4}=\frac{p^2}{4}-q$$

$$\left(x+\frac{p}{2}\right)^2=\frac{p^2}{4}-q$$

$$x+\frac{p}{2}=\pm\sqrt{\frac{p^2}{4}-q}$$

$$x=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}=-\frac{b}{2a}\pm\sqrt{\frac{\left(\frac{b}{a}\right)^2}{4}-\frac{c}{a}}=-\frac{b}{2a}\pm\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}$$

$$\boxed{\frac{b^2}{a^2}\frac{1}{4}=\frac{b^2}{4a^2}}$$

$$=-\frac{b}{2a}\pm\sqrt{\frac{b^2}{4a^2}-\frac{4ac}{4a^2}}$$

$$=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$c\,\frac{p-t}{a-t} = \frac{q-t}{b-t}$$

$$c\,(pb + t^2 - pt - tb) = qa - qt + t^2 - ta$$

e poiché $t = t'$

Se $a = p$ $\quad p = \alpha p + (1-\alpha) t$ $\quad q = \alpha' b + (1-\alpha') t$

$$\rightarrow t = \frac{p - \alpha p}{1-\alpha} = \frac{p(1-\alpha)}{(1-\alpha)} = p$$

$\cancel{q = \alpha p + (1-\alpha) t}$

$\cancel{q = \alpha p + t - \alpha t}$

quindi $\alpha = p = t$

$$p = \alpha p + t - \alpha t$$

$$\boxed{p - t = \alpha (p - t)}$$

$$\alpha = \frac{p-t}{p-t} = \frac{0}{0}$$

$$q = \alpha' b + (1-\alpha') p$$

$$q = \alpha' b + p - \alpha' p$$

$$q - p = \alpha' (b - p)$$

$$\alpha' = \frac{q-p}{b-p} = \frac{p-q}{p-b}$$

$$p = \alpha p + p - \alpha p$$

$$0 = p - p = \alpha p - \alpha p = 0$$

$$\cancel{\alpha = \frac{0}{0}}$$

se $\alpha = 0$, $p = t$

se $p = t$ $\quad \alpha = \frac{0}{0}$

$$\alpha = 0$$

$$p = \alpha a + (1-\alpha) t$$

$$p = t$$

$$t = \alpha a + t - \alpha t \Rightarrow a = t$$

Se $\quad a = q$

$$p = \alpha q + (1-\alpha) t$$

$$t = \frac{p - \alpha q}{1-\alpha}$$

$$p = \alpha q + t - \alpha t$$

$$p - t = \alpha (q - t)$$

$$\alpha = \frac{p-t}{q-t}$$

$$q = \alpha b + (1-\alpha) t$$

$$t = \frac{q - \alpha b}{1-\alpha}$$

$$\alpha = \frac{q-t}{b-t}$$

$$p - \alpha q = q - \alpha b$$

$$p - q = \alpha q - \alpha b$$

$$\alpha = \frac{p-q}{q-b}$$

$$\frac{q-t}{b-t} = \frac{p-t}{q-t}$$

$$q^2 + \cancel{t^2} - 2qt = bp - bt - tp + \cancel{t^2}$$

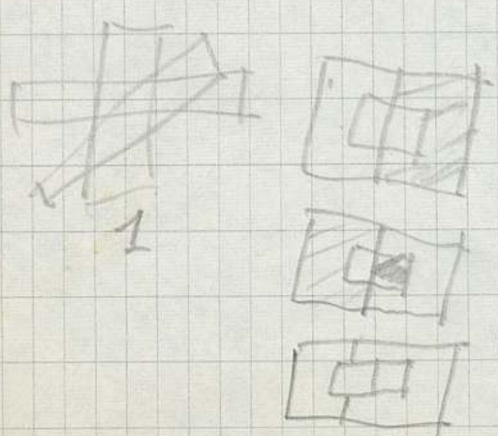
$$tp - 2qt + bt = -q^2 + bp$$

$$t(p - 2q + b) = -q^2 + bp$$

$$t = \frac{-q^2 + bp}{p - 2q + b} = \frac{-q + bp}{p + b - 2}$$

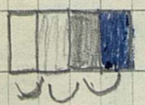
$$t = \frac{aq - bp}{(a+q) - (b+p)} = \frac{q^2 - bp}{2q - b - p}$$

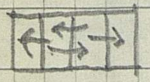
$$= \frac{q^2 - bp}{2 - b - p}$$



1

# Teoria della trasparenza





A B C D

Che cosa importa: l'indipendenza da [illegible] visti della teoria?

Che cosa avviene

a) se si rende impossibile l'uguagliamento BC oppure AB e CD oppure AB o CD

b) se si pone l'alternativa fra BC e (AB, CD)

c) se si rende più forte la forza di coesione degli uni o degli altri

a) 1. AB e CD molto simili cromaticamente, BC dissimili
2. Viceversa
3. AB simili, BC e CD dissimili — Colori
4. AD dissimili, BC e CD simili

Tutto questo alternato con l'appartenenza figurale (utilizzare Gottschaldt)

b) con colori ~~B~~ $B - A > C - D$

N S C B

$S - N = C$

~~B + B = NC~~

~~C + A = C~~

Se $\begin{cases} N + C = S \\ B + C = CC \end{cases}$ allora $\dfrac{N \; S \; CC \; B}{N \; \overset{C}{N} \; \overset{C}{B} \; B}$ 23)

A prediction formula for phenomenal transparency

ombre per variare

$a \quad p \quad q = p - x \quad b = a - x$

$$\alpha = \frac{p-(p-x)}{a-(a-x)} = 1 \qquad t = \frac{aq-pb}{(a+q)-(p+b)} = a(p-x)-p(a-x)$$

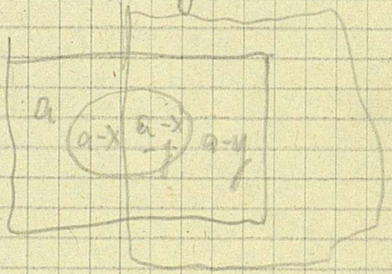
$$= \frac{ap - ax - ap + px}{\cancel{a} + \cancel{p} - \cancel{x} - \cancel{p} - \cancel{a} + \cancel{x}} = \frac{px - ax}{0}$$

$p = a - x \quad q = b - x$

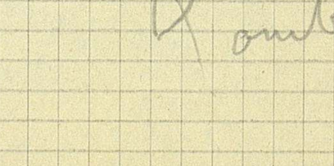
$$\alpha = \frac{\cancel{a} - \cancel{x} - b + \cancel{x}}{a - b}$$

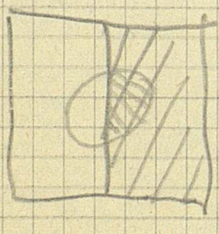
~~p = a~~

$p = a - x \quad q = a - x - y \quad b = a - y$



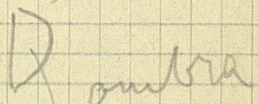

$$\frac{\cancel{a} - \cancel{x} - \cancel{a} + \cancel{x} + y}{\cancel{a} - \cancel{x} + y}$$



$a \quad p \quad q = b - x \quad b$

$$\frac{p - b + x}{a - b}$$



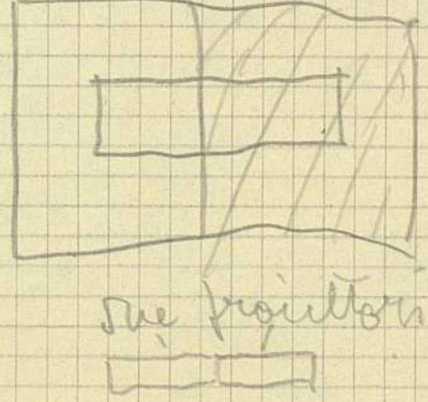
1 Episcotista - illuminaz. uguale epist. e sfondo. Variazione corr. al triangolo di Rimondini, tenendo A e B costanti (non bianco e nero soltanto)

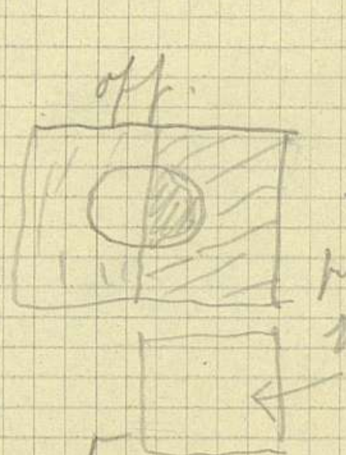
$p = \alpha a + (1-\alpha) t \quad a$

2. Episcotista e ombra sullo sfondo

$$p = \alpha (a-x) + (1-\alpha) t \qquad \frac{p - \alpha(a-x)}{\cancel{1-\alpha}} = \frac{q - \alpha(b-x)}{\cancel{1-\alpha}}$$
$$q = \alpha (b-x) + (1-\alpha) t$$
$$p = q - \alpha(b-x) + \alpha(a-x)$$
$$p - q = \alpha[(a-x) - (b-x)]$$
$$\frac{p-q}{a-\cancel{x}-b+\cancel{x}} \qquad t = \frac{a(q-x)-(p-x)b}{a+(q-\cancel{x})-p+\cancel{x}-b}$$

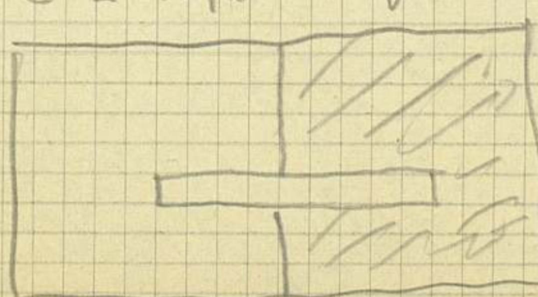
3. Proiezione sui due sfondi di due rettangoli luminosi

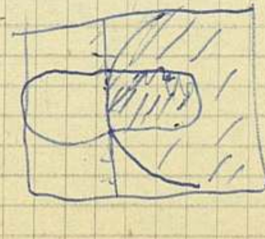


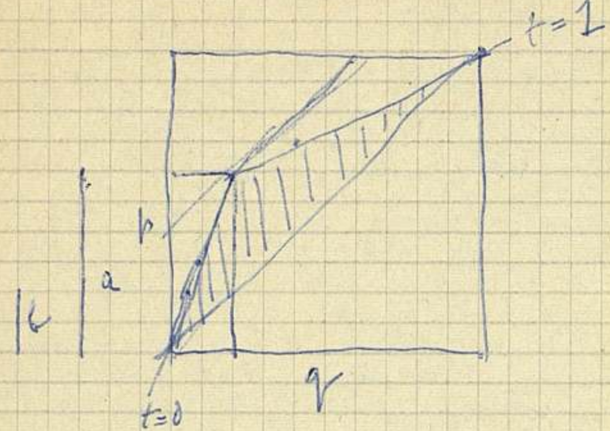




e finestra
più [illegible] luce
nel mezzo

4. Foro e 2 [illegible] di [illegible]








$t=0$

$aq = bp$

$t=1$

$\overset{4}{y}\overset{4}{q} - x\overset{2}{b} = \overset{4}{y} - x$

$x - xb = y - yq$

$x(1-b) = y - yq$

$x = \dfrac{y-yq}{1-b} = .4 - .4.4 = \dfrac{.24}{.8} = .3$

$p - q = .3$

$p = .3 + .4$

$x = .4 \quad y = .4$

$q = .3 \quad p = .7 \quad y =$

controllo

$\dfrac{yq - xb}{x - y} \quad 0$

Bibliografia
per l'esperimento con plastica grigia

Vedere Helmholtz e Hering in Fuchs

separare le pagine!

Nuova copia con spazi 4 x 4 per le figure

e V. Schumann

F. METELLI

(Padova, Italy)

A RESEARCH ON THE PERCEPTION OF TRANSPARENCY

~~(Summary)~~

In the research on Perception, important results have been reached through the study of "illusions", namely phenomena where perception fails to give correct information about the objective situation.

In the case of the perception of transparency, illusions help us to "see" the very problem. In fact, under normal conditions, where we are perceiving as transparent what is permeable by luminous radii, there seems not to be any perceptual problem. But if we put a sheet of transparent plastic, or a piece of glass on an homogeneous background, we do not perceive it as transparent, while in situations like Fig. 1, where there is no physicall transparency, we perceive transparency (1); hence it is natural to ask oneself the reason for these facts.

Fig. 1

---

(1) It could be objected that in this case, as in all the examples given in this paper the impression of transparency is incomplete, poor, and ~~yet~~ however different from a "true" transparency, as perceived when we look through a (coloured) glass of a window, or through the surface of the sea.

The reason for this difference is that in our pictures three-dimensionality is lacking, which is not a necessary, but a very powerful condition supporting the impression of transparency. But comparing real (= physical) transparency with apparent transparency un

./.

An initial explanation can be attempted in terms of cues: normal cues of transparency are lacking in the first case and present in the second (a surface seen through a film appears altered in its colour, because of absorbtion and reflection by the transparent surface). But with irregular figures, like Fig. 2 and 3, it is by no means clear why we perceive in Fig. 2 two surfaces one seen through the other, and not several juxtaposed blots as in Fig. 3.

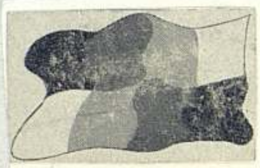


Fig. 2

Fig. 3

In general terms, the problem of perceptual transparency can be stated as follows: under which conditions the stimulation of a group of visual receptors of the retina gives rise to the perception of two surfaces, one of which is seen through the other?

This problem has been evidenced and discussed by ~~H.~~ Helmholtz and ~~E.~~ Hering,(1) and further subjected to experimental researches by Fuchs, Tudor-Hart, Heider, Koffka, Metzger, Kanizsa, Metelli.

It is easy to show that there are two orders of conditions of perceptual transparency, figural and colour-conditions: ~~because~~ in fact it is possible to abolish perceptual transparency by altering form

---

der the same conditions of bidimensionality (that is, comparing a chess board ~~like Fig. 2~~ like Fig. 4 with a film of grey transparent plastic on it, with a formally and chromatically identical model, but constructed as a mosaic by juxtaposition of 4 different shades of grey opaque paper) the impression is the same, and nobody can say where there is physical transparency and where not.

(1) H. Helmholtz - Physiologische Optik, p. 313
E. Hering -

(Fig. 2,3) or colour (Fig. 4,5).

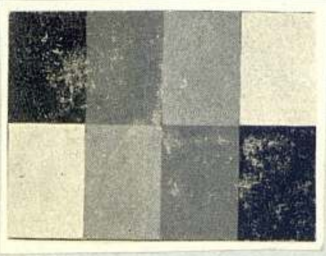


Fig. 4

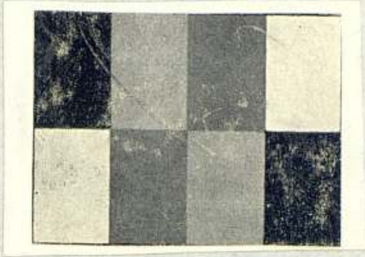


Fig. 5

The subject of this paper is a research into the colour conditions in perceptual transparency.(1) As ~~my~~ the purpose ~~has been~~ of this study was to find a quantitative law about the influence of colour on the perception of transparency, ~~my~~ the first task has been to choose a quantitative expression of colour.

It is well known that for the definition of a colour, not less than 3 numbers are needed, while for the definition of the different shades of grey, from white to black (which are named achromatic colours) only one number is needed, namely the index of reflectance or albedo, the amount of light reflected from a unit area divided by the amount of light it receives (the formula being $L = \frac{i}{I}$, where L stands for the coefficient of reflection or albedo, i stands for the intensity of reflected light and I for the intensity of light falling into the area). Since an absolute white reflects the whole amount of light that it receives, while the absolute black absorbs the whole light falling on it, the absolute white has reflectance 1, the absolute black, reflectance 0 and the various shades of grey have coefficients of reflectance between 0 and 1.

For this reason - the achromatic colours being univocally defined by one number, ~~I found it~~ it appeared suitable to begin by confining my study to the field of acromatic colours, which from now on will be named simply colours.

Let us start from a simple figure (Fig. 6) where subjects normally perceive transparency. The figure has been chosen because it

---

(1) This problem has been considered from a special point of view in F. Metelli - Zur Analyse der phänomenalen Durchsichtigkeitserscheinungen (Gestalt und Wirklichkeit, Festgabe für F. Weinhandl, Berlin, 1967), that is chiefly devoted to the theory and to a first analysis of the figural conditions of transparency.

Metzger (Gesetze des Sehens, p. – ) ~~has been~~ has given the striking demonstration that ~~a~~ ~~then~~ an effect of phenomenal transparency can be produced by juxtaposing pieces of opaque cardboard.

In fact, all the figures of this paper can be objectively described as models where perceptual transparency is obtained by juxtaposition of opaque surfaces reproduces the situation of transparency obtained by an episcotister - a rotating wheel with open sectors -; the following considerations stand for both situations, but they are easier to follow if we start with a model where ~~perceptual~~ perceptual transparency is obtained ~~by Metzger's~~ by the method of juxtaposition of opaque surfaces.[1]

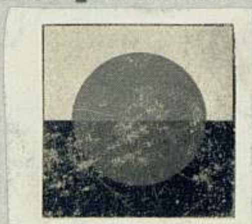


Fig. 6

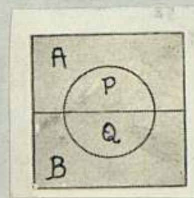



Fig. 6a

In this figure - which can be considered as a general model for perceptual transparency phenomena - we distinguish 4 different regions, with four different shades of grey; we name them A P Q B (capital) and the respective reflectances a p q b (small). The stimulation originated by the P region produces 2 different perceptual effects: we see an anterior layer T, which is transparent, and through this a second layer, the latter being of the same colour as the contiguous region A. (The same observation can be made for the region Q, but for the moment, let us confine our argument to P) (2). Therefore the perceptual phenomenon of transparency has been described as a case of perceptual scission - one of the much studied scission-phenomena where one sort of stimulation produces

(1) ~~This method is due~~ to W. Metzger (Gesetze des Sehens, II ed. 1953 pp 127-8)

(2) It has to be stressed that every figure in this paper can be described according to the preceding scheme: there are always two non-splitting regions, A and B, which constitute the ground, and two splitting regions, P and Q, P splitting into A and T, and Q splitting into B and T. One needs only to add that most figures are serial repetitions of the sequence APQB alternated with the inverse sequence BQPA. (See for ex. Fig. 1,2,4 where two parts can be distinguished, an upper and a lower part, where the two inverse sequences are easy to identify; and Fig.8-15, where several horizontal sequences are identifiable).

two effects, as for example surface-colour and illumination (1).

At this point it is natural to ask oneself what is the relation between stimulus colour and scission colours. In our figure we can easily perceive them all: the stimulus colour if we isolate the P region, and the scission colours when we perceive transparency.

A simple solution to the above problem is due to G. Heider and K. Koffka, who stated, and gave experimental proof that scission colours are such that, mixed together according to Talbot's law, reproduce the stimulus colour.

The example given by Koffka in his treatise (2) is as follows. If the stimulus colour is grey, and the conditions require that one of the scission colours be blue, then the other scission colour must be yellow. (3) Symbolically, if Y + B = G, then G - B = Y.

Heider and Koffka's theory is the starting point of this research.

It is clear that Heider and Koffka's formulation is not an algebraic formulation of the problem, because G, Y and G do not symbolize numbers. But it is possible to give an algebraic formulation to the problem if we confine ourselves to acromatic colours, ~~which~~ using reflectance as their measure. ~~can each of them be univocally defined by the measure of their reflectance.~~

My line of reasoning has been the following. If according to Heider and Koffka's statement, the same law (that is Talbot's law

(1) In fact, in this case also, as in the colour constancy phenomenon, if we isolate the region which is represented twice (that is, in Fig. 6, the circle, which is perceived as a transparent layer and as a part of the ground seen through it) the perceptual scission disappears, and we perceive only one layer.

(2) K. Koffka - Principles of Gestalt Psychology, New York 1935, pp. 260-264

(3)

in the special case of mixture through the colour wheel (1)) rules colour-mixture and colour scission, then it is possible to use the law of colour mixture for an algebraic description of colour-scission.

Talbot's law says that if two acromatic colours, whose reflectances are $a$ and $b$ are mixed in equal quantities, the reflectance of the mixture $c$ is the arithmetic mean of the two colours, $c = \frac{a+b}{2}$. If the colours $a$ and $b$ are mixed in quantities $m$ and $n$ ($m$ being the quantity of $a$ and $n$ the quantity of $b$ ) the reflectance of the mixture $c$ is

$c = \frac{ma + nb}{m+n}$ or, in other words, the reflectance of the mixture-colour is the weighted average of the components, the weights being the quantities of the components.

The same formula can be expressed in a more suitable form if instead of the absolute quantities, we use as weights the proportions (summing up to 1) of the components. Putting $\frac{m}{m+n} = \alpha$ and $\frac{n}{m+n} = 1 - \frac{m}{m+n} = 1 - \alpha$ the above equation takes the form

$$c = \alpha a + (1 - \alpha)b$$

$\alpha$ and $1 - \alpha$ being the proportions in which the components are present in the mixture.

But if Talbot's law rules both phenomena, colour mixture and colour scission, then the above equation, describing colour mixture, describes also colour scission. Since it is convenient to use symbols according to the preceeding example, the colour scission ~~or transparency~~ equation will be used from now on in the form

$$p = \alpha a + (1 - \alpha)t,$$

(1) According to W. Ostwald (Mathetische Farbenlehre (Leipzig 1921, p.131 ~~p. 13?~~ ) the law of colour mixture goes back to Newton).

where p is the reflectance of the stimulus-colour, a the reflectance of the second layer, t the reflectance of the first and transparent layer, and $\alpha$ and $(1-\alpha)$ are the quantities or more exactly the proportions into which the stimulus colour has been divided in the two layers (1).

But what meaning is to be given to quantities in this case? Or rather how will perceptually different quantities of colour appear, when distributed to equal surfaces?

Different quantities distributed to equal surfaces can give as a result only a difference in colour density. And on the first layer a difference in colour density can appear only as a difference in transparency, while on the second layer it can reveal itself only through a difference in intensity which is correlative to the transparency of the first layer. In other words, great density on the first layer and little density on the second means little transparency of the first and little intensity or little visibility of the second layer; while less colour on the first layer means little density or great transparency, and more colour on the second layer means great intensity and great visibility.

Let us check this interpretation with the equation. What happens if $\alpha = 0$, that is, if the first layer T gets the whole colour of P ~~p~~? In this case the equation reduces to $p = t$, and ~~a~~ the second layer, having no colour at all, is not visible: we see only the first layer T ~~t~~, which is completely opaque. Thus $\alpha = 0$ defines the limiting case of the perception of an opaque surface on the top of another surface.

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(1) It is perhaps opportune to remember that ~~the colour of~~ the P region is divided into two layers, A and T, and the surface of each one is egual to the surface of P.

And what happens if $\alpha = 1$? In this case it is the second layer which gets the whole colour (the solution of the equation being $p = a$) and the first layer, having no colour, disappears completely. This means that the first layer is perfectly transparent and therefore wholly invisible.

In both these cases ($\alpha = 0$ and $\alpha = 1$) there is no colour scission and obviously no perception of transparency: colour scission ~~begins~~ takes place only in the intermediate cases, that is when $\alpha$ is between 0 and 1.

Which then is the meaning of the coefficient $\alpha$, which has a maximum value 1 when the transparency is perfect, a minimum value 0 when there is no transparency at all, and has a high value when a little quantity of the scission-colour comes to the first layer, and therefore the transparency is great, and a low value if a great quantity of the scission colour comes to the first layer, and therefore the transparency is little?

The obvious inference seems to be that $\alpha$ - which measures the proportion of colour going to the second layer seen through the transparent one (the intensity or clearness with which the second layer is perceived) - measures the degree of transparency and is therefore a coefficient of transparency. But quantitative experiments show beyond any doubt that transparency depends also on the chromatic quality of the first layer, and therefore $\alpha$ is only a factor of transparency. ~~The~~ A second factor is phenomenal colour, that is colour measured by the logarithm of the reflectance (1).

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(1) All other conditions being equal, the transparency of an achromatic surface is an inverse function of the whiteness (or the reflectance) of the surface. The inverse relation between reflectance and phenomenal transparency can be easily shown using episcotisters of equal opening and different shades of grey.

(1) Experimenting with an episcotister it is easy to show that if the colour a (b) of the ground ~~a (b)~~ and the colour t of the rotating sector are kept constant, the transparency is a direct function of $\alpha$, the open sector measured as a proportion of $360^\circ$ ($\alpha = \frac{360 - \varphi}{360}$, $\varphi$ being the size of the rotating sector)

(2) This sentence ~~(has to be referred to)~~ refers to the transparency cases experienced in everyday ~~of the common~~ life or to the models discussed here like fig 1, 2, 4, 6 etc. In these cases ~~..~~ we can measure the reflectances of the regions A and P (B and Q), while we don't know $\alpha$ and the reflectance of the transparent layer T. In the special case of transparency obtained through an episcotister the known terms of the problem are the colour of the ground a (b), the colour ~~of~~ t of the rotating sector, and the size of the ~~secto~~ open sector $\alpha$.

The other conditions (first of all (a and t) colours (1)) are

Thus $\alpha$ is the index of phenomenal colour-scission; it acquires the function of a coefficient of transparency only if ~~colour p is~~ held constant. And, phenomenal colour scission being a necessary condition of transparency, the latter is possible only if $\alpha$ is less then 1 and more then 0.

With this definition of $\alpha$, the above equation should not contain any unknown symbol, as a and t are the reflectance coefficients which measure the colours of the scission layers. There is, however an important difference between a and t: a is the colour of the contiguous region A, and is therefore a known quantity (one of the known terms of the problem), while t, together with $\alpha$, are the unknowns of the equation.(2)

which identifies the reflectances, that is the measures of achromatic colours

In fact the problem of phenomenal colour scission can be given the following formulation: starting from the knowledge of the colours (or reflectances) of the regions A and P, is it possible to predict the proportion $\alpha$ in which the colour p is divided among the two layers, and the colour of the transparent layer t?

The answer is of course a negative one, bacause the unknowns are two, and therefore the equation is indeterminate.

It is perhaps suitable to look more deeply into the question, because it can seem that, when the quantity p is divided into a and t, if p and a are known, t is determined.

A way of clarifying the question is to begin by putting it into very simple terms. If p is divided into equal parts, the equation simplifies to $p = \frac{a+t}{2}$ or $p = \frac{1}{2}a + \frac{1}{2}t$, and in this case once a is chosen, t is determined. If p is a point on a segment which represents the sequence of numbers from 0 to 1, a and t have to be on opposite sides of and at the same distance from p. That is, p splits into two colours, each of which is half the quantity of p, and which, mixed together once again give p (the same colour in the same quantity).

But following the above line of reasoning we have renounced to one dimension of change, by fixing the proportion of splitting as 0,5 to 0,5. If we choose another splitting ratio, e.g. 0,25 to 0,75, and again consider p as a point on the 0-1 segment, a and t have also to

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(1) Experimenting with an episcotister it is easy to show that if the colour a (b) of the ground and the colour t of the rotating sector are kept constant, the transparency is a direct function of $\alpha$, the o p e n sector measured as a proportion of 360° ($\alpha = \frac{360-\varphi}{360}$, $\varphi$ being the size of the r o t a t i n g sector).

(2) This sentence refers to the transparency cases experienced in everyday life or to the models discussed here like fig. 1, 2, 4, 6 etc. In these cases we can measure the reflectances of the regions A and P (B and Q), while we do not know $\alpha$ and the reflectance of the transparent layer T. In the special case of transparency obtained through an episcotister the known terms of the problem are the colour of the ground a (b), the colour t of the rotating sector, and the size of the open sector $\alpha$.

(that is, his position on the 0-1 scale of the reflectances)

be on opposite sides, but not at the same distance from p: the distance of t from p has to be 1/3 the distance of a from p. Is means that if p and a are fixed, the colour of t depends on the splitting ratio of p~~, and what is more interesting, there is compensation between splitting ratio and colour of the transparent layer.~~

But so far we have used only one half of the A P Q B model (Fig. 7) and thus only one half of the data. In fact we can write a second equation using b and q, that is

$$q = \alpha' b + (1 - \alpha')t'$$

and if we may put $\alpha = \alpha'$ and $t = t'$, which seems to be very often, if not always, right (1), the system of two equations with two unknowns is soluble. The solutions are

$$\alpha = \frac{p-q}{a-b} \qquad t = \frac{aq - bp}{(a+q) - (b+p)}$$

At this point it seems opportune to check the deduced formulas, that is to see if and to what extent there is a correspondence between theoretically deduced formulas and the facts.

Let us begin with the equation of the phenomenal scission index $\alpha$. The equation defines the field of transparency, because transparency is possible only for the values of $\alpha$ between 0 and 1; because as we have already seen $\alpha = 0$, (perfect opacity) and $\alpha = 1$ perfect transparency) are the limiting cases where phenomenal scission is lacking, and therefore not to be considered among transparency

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(1) There are cases where the perceived transparency of P and Q is not the same. As the colour of the transparent layer T appears the same on both regions (that is $t = t'$) it seems reasonable to hypothesize $\alpha \neq \alpha'$, namely that in these cases the quantity of colour assigned to T in the splitting process is different for the part of T covering A from that covering B. [~~But it is also possible that the difference in transparency is due to the difference in colour between A and B.~~]

phenomena. $\alpha > 1$ and $\alpha < 0$ would mean that one or the other layer would receive a negative quantity of colour, a situation which is devoid of meaning.

Therefore two necessary conditions follow from the formula

1. $|a-b| > |p-q|$ (otherwise $\alpha \geqslant 1$)

2. $(a > b) \iff (p > q)$ (otherwise $\alpha < 0$)
   $(a < b) \iff (p < q)$

The first condition says that the difference between the reflectances of the regions A and B has to be greater than the difference between the reflectances of the splitting regions P and Q (1). Negatively expressed it becomes a sufficient condition: if the difference in reflectance between the splitting regions P and Q is greater than the difference between the non splitting regions A and B, there cannot be transparency.

Fig. 7 Fig. 8 Fig. 9

This condition can be easily controlled. Fig. 6 and 7 show two identical models with the only difference, that in Fig. 6 the difference in reflectance between the internal semicircular regions is clearly less than the difference in reflectance between the external regions, while in Fig. 7 the contrary in true. The

(1) As measures of reflectances are higher for lighter than for darker shades of grey, in terms of (achromatic) colour the $>$ symbol means "brighter than", and the $<$ symbol "darker than".

result is that in Fig. 6, and not in Fig. 7 transparency is perceived. It is interesting to see what happens in another sample of figures (Fig. 8 and 9) which are constructed following the same principles (1). In this case both figures can be perceived as transparent, but different regions are transparent in each figure; that is, the contiguous regions where the splitting phenomenon takes place are always the regions between which there is less difference in reflectance. In this case the relation $|a-b| > |p-q|$ has decided which regions take the functions of p and q, (that is, become the splitting regions) and which take the functions of a and b. Therefore in this case also, the above necessary condition has been respected (2).

The second condition can be espressed by saying that the brightness gradient (or the fall of brightness-level) between p and

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(1) Fig. 8-15 are alternations of the sequences B A P Q B A and A B Q P A B. The critical sequences, necessary for the arousal of perceptual transparency are, of course A P Q B and (or) B Q P A. The addition of the squares A and B at the beginning and at the end of the sequence has only the effect of rendering coercive and therefore general among subjects the perceptual organisation which gives rise to phenomenal transparency.

(2) This is, of course, only an initial and very rough check. Other checks can be done constructing models where $|p-q|$ approaches $|a-b|$. In these cases the perception becomes ambiguous: there is alternation between transparency of the central region (like Fig. 8) and transparency of the peripheral region (Like Fig. 9). In other words the law $|a-b| > |p-q|$ holds only when the difference between $|a-b|$ and $|p-q|$ reaches a certain size.

q on the one hand and between a and b on the other hand must have the same direction (1).

This condition also may be checked. On Fig. 4, where transparency is generally perceived, this condition is respected, because the splitting region which is contiguous to the brightest region A, the P region, is clearer than the other splitting region, Q. If we reverse the brightness gradient between P and Q (Fig. 5) transparency is no longer perceived.

Fig. 10 Fig. 11

Besides other, less interesting necessary conditions, it can be derived from the splitting-index equation that if the colour t of the transparent layer T, is held constant,[2] when the difference between a and b is much greater than the difference between p and q, there is little transparency (Fig. 10), whereas when the difference between a and b is hardly greater than the difference between p and q, transparency is great (Fig. 11).

The other formula, $t = \frac{aq - bp}{(a+q) - (b+p)}$ is more complicated and does not offer the opportunity of deriving simple predictions. Nevertheless a way has been found to derive qualitative predictions also

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(1) The formulation can be simplified to $(p > q) \Longleftrightarrow (a > b)$ or, yet further to $p > q$ if we define $a > b$, or, in other words, the brighter of the two shades of grey which form the ground, is named a.

(2) The necessity of keeping constant the colour of the transparent layer T depends on the formerly stressed fact that colour itself is a factor of transparency.

about the colour t of the transparent layer.

The original equation $p = \alpha a + (1-\alpha)t$ can be given the form $\alpha = \frac{p-t}{a-t}$. As it has already been said, in the case of transparency we have $0 < \alpha < 1$.

Let us consider first the disequation $\alpha > 0$, and therefore

$$1. \quad \frac{p-t}{a-t} > 0$$

This condition implies that numerator and denominator of the fraction are either both positive or both negative. We have therefore to take into consideration two cases.

CASE 1A

As numerator and denominator are positive, we have

$(p-t) > 0, \quad (a-t) > 0$

therefore $p > t$ and $a > t$

or

if $p > t$ then $a > t$ and viceversa

or

$$\boxed{(p > t) \Longleftrightarrow (a > t)}$$

$(p > t)$ implies and is implied by $(a > t)$

CASE 1B

As numerator and denominator are negative, we have

$(p-t) < 0, \quad (a-t) < 0$

therefore $t > p$ and $t > a$

or

if $t > p$ then $t > a$

or

$$\boxed{(t > p) \Longleftrightarrow (t > a)}$$

$(t > p)$ implies and is implied by $(t > a)$

Now let us consider the second disequation, namely $\alpha < 1$ and therefore

$$2. \qquad \frac{p-t}{a-t} < 1$$

with reference to both cases A and B.

CASE 2A

As (a-t) is positive, multiplying both members of the disequation by (a-t), the direction of the disequation remains unchanged

$$\frac{p-t}{\cancel{a-t}}\,\cancel{(a-t)} < 1\,(a-t)$$

that is $(p-t) < (a-t)$

and therefore $\boxed{p < a}$

CASE 2B

As (a-t) is negative, multiplying both members of the disequation by (a-t), the direction of the disequation changes

$$\frac{p-t}{\cancel{a-t}}\,\cancel{(a-t)} > 1\,(a-t)$$

that is $(p-t) > (a-t)$

and therefore $\boxed{p > a}$

ASSOCIATING 1A and 2A

namely from the hypothesis that numerator and denominator are positive, that is

from $(p > t) \iff (a > t)$

and ~~a>~~ $p < a$

it follows that

$$\boxed{a > p > t}$$

ASSOCIATING 1B and 2B

that is, from the hypothesis that numerator and determinator are negative, that is

from $(t > p) \iff (t > a)$

and $p > a$

it follows that

$$\boxed{t > p > a}$$

Till now the consequences which followed from the above formula prove only what was a natural experctation, namely that when a phenomenal scission occurs, if one of the scission colours a or t is brighter than the stimulus colour p, then the other scission colour (t or a) is necessarily darker.

But so far we obtained the above inferences only about the areas A and P. Obviously, following the same line of thought, the same algebraic relations are obtained about the areas B and Q.

The conditions are, therefore, for the areas A and P

A. $a > p > t$ or B. $t > p > a$

and for the areas B and Q

C. $b > q > t$ or D. $t > q > b$

Putting together each of the alternative conditions for the areas A and P with each of the alternative conditions for the areas Q and B, the following combinations are obtained.

| AC | AD | BC | BD |
|---|---|---|---|
| $a > p > t$ | $a > p > t$ | $t > p > a$ | $t > p > a$ |
| $b > q > t$ | $t > q > b$ | $b > q > t$ | $t > q > b$ |

The meaning of this operation is that by choosing a given brightness order for the regions A P Q B, we can predict the place of t, the colour of the transparent layer, on the above sequence, or brightness scale.

Of course, the occurrence of phenomenal transparency is not guaranteed by these combinations, because transparency conditions were taken into account for every half figure separately and not for the figure as a whole. Therefore, if the previously inferred necessary chromatic conditions of transparency are lacking, transparency cannot be perceived. The meaning of the combined brightness sequences is, therefore, that <u>if transparency occurs, then, from the brightness relations between the areas, the degree of brightness of the transparent layer can be predicted.</u>

From the preceeding analysis three different cases follow

1. The transparent layer T is the darkest of the sequence (case AC). If $a > p$ and $b > q$, $t$ is the darkest of all. Combining conditions A and C, two sequences, $a > p > b > q > t$ (Fig. 12) and $a > b > p > q > t$ (Fig. 13) are generated which do not conflict with the necessary conditions of transparency (1).
2. The transparent layer T is darker than $a$ and $p$ and brighter than $q$ and $b$ (cases AD and BC (2)). The sequence obtained by combining conditions A and D is $a > p > t > q > b$ (Fig. 8, 10, 11) which does not conflict with the necessary conditions of transparency.
3. The transparent layer T is the brightest of the sequence (case BD). If $p > a$ and $q > b$, $t$ is the brightest of all. Combining conditions B and D, two sequences, $t > p > a > q > b$ (Fig. 14) and $t > p > q > a > b$ (Fig. 15) are generated, which do not conflict with the necessary conditions of transparency.

Fig. 12 Fig. 13 Fig. 14 Fig. 15

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(1) $a > p$ and $b > q$ can also be combined to a sequence $a > b > q > p$, which would conflict with the necessary condition $(a > b) \Leftrightarrow (p > q)$ and in fact in this case we do not perceive transparency. The combination $a > (p=b) > q > t$ (where there are only 3 different shades of grey) does not conflict with the necessary conditions of transparency: in fact under these conditions transparency can be perceived.

(2) Case BC repeats case AD if $a$, instead of being the brightest, is the darkest of the four regions.

Fig. 12, 13, 8, 14, 15 that correspond to cases 1a, 1b, 2, 3a, 3b can be considered an initial check of the above deductions.

The theory can be applied also in special cases, when the different shades of grey are 3 instead of 4 (if $a=p$ or $a=q$; $a=b$ and $p=q$ being excluded because in this case $\alpha$ is outside the field of validity); and also to the cases of more then 4 different fields of grey.

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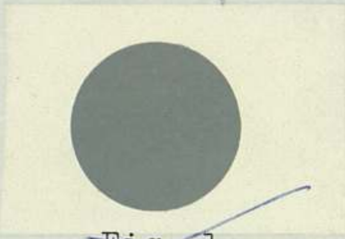
# A RESEARCH ON ~~PERCEPTUAL~~ The perception of TRANSPARENCY

(Summary)

In the research on Perception, important results have been reached through the study of illusions, namely phenomena where perception fails to give correct information about the objective situation.

In the case of the perception of transparency, illusions help us to "see" the very problem. In fact, under normal conditions, where we are perceiving as transparent what is permeable by luminous radii, there seems not to be any perceptual problem. But in Fig. 1 where there is no perception of transparency although the circle of plastics is physically transparent, and in Fig. 2 where subjects commonly have the impression of transparency although being aware that there is no physical transparency it is natural to ask oneself the reason for these facts.



~~Fig. 1~~

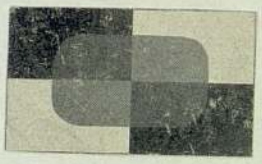


Fig. 1

An initial explanation can be attempted in terms of cues: normal cues of transparency are lacking in the first case and present in the second (a surface seen through a ~~smoked glass~~ film appears ~~less bright~~ altered in its colour, because of absorbtion ~~of a part of the luminous radii reflected by the surface~~ and reflection by the transparent surface, and reflection by the transparent film surface). But with irregular figures, like Fig. 2 and 3, it is by no means clear why we perceive in Fig. 2 two surfaces one seen through the other, and not several juxtaposed blots as in Fig. 3; ~~or why exchanging colours in the~~

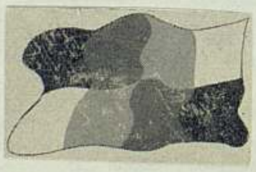


Fig. 2

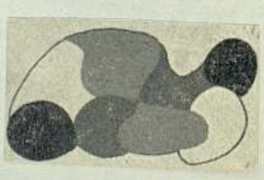


Fig. 3

In general terms, the problem of perceptual transparency can be stated as follows: under which conditions the stimulation of a group of visual receptors of the retina gives rise to the perception of two surfaces, one of which is seen through the other?

This problem has been evidenced (and discussed) by H. Helmholtz and E. Hering, and

⑥ But if we put a sheet of transparent plastic, or a piece of glass on an homogeneous background, ~~(Fig. 1 is a~~ we don't perceive it as transparent; ~~(Fig. 1 is a circle photography of a circle of transparent gray plastics on a homogeneous white background)~~; while in situations like Fig 1, where there is no physical transparency ~~subjects~~ we have the impression of transparency(1); ~~Fig 2 is commonly described as a transparent surface chessboard partly covered by a transparent gray film)~~. Hence: it is natural to ask oneself the reason for these facts.

(1) It could be objected that in this case, as in all the examples given in this paper the impression of transparency ~~has not the~~ is ~~an~~ incomplete, poor, and however different from a "true" transparency, as perceived ~~if~~ when we look through the ~~water~~ through a (coloured) glass of a window, or through the ~~the~~ surface of the sea.

The reason of this difference is that in our pictures, which is not a necessary, but a very powerful condition ~~which favours~~ supporting the impression of transparency, tridimensionality is lacking. But comparing real (= physical) transparency with apparent transparency under the same conditions of tridimensionality (that is, comparing ~~the reproduction of picture with fig. 2 or 3, with the~~ a chessboard like fig. 2 with a film of grey transparent plastics ~~with~~ ~~an equally~~ a formally and chromatically identical model, but constructed ~~consisting~~ as a mosaic by juxtaposition of ~~4~~ different shades of grey opaque paper) the impression is the same, and nobody can say where there is physical transparency and where not.

further subjected to experimental researches by Fuchs
Tudor-Hart, Heider, Koffka, Metzger, Kanizsa, Kutelin,



~~checkboard we perceive four rectangles seen through a transparent surface in Fig. 5, and 8 rectangles in Fig. 6.~~

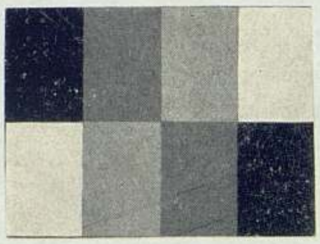


Fig. 4

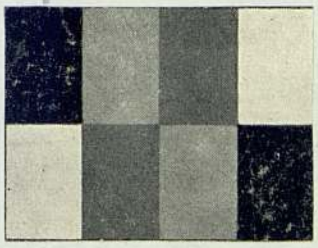


Fig. 5

The examples show that perceptual transparency depends on two orders of conditions, figural and colour conditions. Colour is here to be understood in the wide meaning, which includes, as colours, also the different shades of grey (acromatic colours). I shall report here how starting from a theoretical point of view I tried to deduce laws about colour conditions for transparency.

The perceptual phenomenon of transparency may be described as a chromatic scission. Whenever we perceive transparency there is a region which is represented twice, like two strata with two different colours or shades, one of which is seen through the other. But if we isolate this region, we perceive only one layer; and the colour of this layer corresponds roughly to the stimulation of the region of the retina on which it projects its image.

Facts may be described as follows (Fig. 7 and 8): there is

Fig. 6

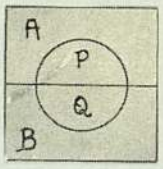



Fig. 6a

a retinal stimulation p, and while in general this stimulation causes the perception of a surface P, under special conditions it gives rise to two perceptual objects, a transparent surface T and behind this an opaque surface A. What will the precise relation be between the three colours p, t, a of the surfaces P, T, A ?

There is a law - which goes back to Newton - describing the opposite phenomenon, namely not the scission phenomenon $p \rightrightarrows {t \atop a}$, but the fusion phenomenon ${a \atop b} \rightrightarrows c$. The existence of this law (1) ~~(Talbot's law)~~ suggested the hypothesis that as it predicts the result of fusion of two colours, it could also rule the opposite field of colour scission, that is the field of transparency, allowing the prediction of the result of colour scission.

For simplicity we have to confine ourselves to the case of achromatic colours, that is of the different shades of gray, from black to white; these are measured by the index of reflectance or albedo, which gives 1 for absolute white, 0 for absolute black, and the numbers between 1 and 0 for the different shades of grey.

Talbot's law is expressed by the equation

$$\alpha a + (1 - \alpha)b = c$$

where a and b are the starting colours (measured by their albedo), $\alpha$ and $(1 - \alpha)$ the proportions in which the starting colours are taken, and c, the result of the mixture.

If the hypothesis is right, the same equation, read in the opposite direction, should describe quantitatively the splitting of the colour p into the colours a and t : $p = \alpha a + (1 - \alpha)t$

At first glance, the hypothesis seems to be unfruitful, because while in the case of the fusion there is only one unknown, namely the albedo of the fusion colour c, in the case of splitting there are three unknowns, a, t and $\alpha$. But it is easy to note that a is the same colour as the protruding (and directly measurable) part of the object which is localised behind the transparent layer; so the unknowns are only $\alpha$ and t.

Therefore, setting up a second equation, with the same unknowns, which is possible, making certain allowances, the system is soluble, and the solutions allow us to check the hypothesis.

Perceptual transparency is the impression of ~~seing~~ seeing through a medium or an object. It will be shown that this phenomenon dipends on special conditions; and these are not the conditions of physical transparency, namely permeability by luminous radii. As a matter of fast, physical transparency is neither a necessary nor a sufficient condition of perceptual trahsparency, because There are situations where the re is physical but not perceptual transparency, and on the contrary, situations were there is perceptual but not physical transparency (Fig. 1, 2 ).

It is easy to show that there are two orders of perceptual conditions of transparency, figural and color-conditions, because it is possibile to abolish perceptual transparency either altering form (Fig. 2, 3) or color. (Fig. 4, 5)

The subject of this ~~lecture~~ paper is a research ~~about~~ into the color conditions in perceptual transparency. As my pourpose has been to find a quantitative law about the influence of ~~color on~~ the perception of transparency, ~~You will judge if and till where I have been suc cessful.~~

My first task has been to choose a quantitative expression of colour.

grey

in the special case of ~~fusion~~ mixture through the colour wheel (1)

law) rules colour-mixture and colour scission, then it is possible to use ~~Talbot's law~~ The law of colour mixture for an algebraic description of colour-scission.

Talbot's law says that if two acromatic colours, whose reflectances are a and b are mixted in quantities m and n (m being the quantity of a, and n the quantity of b) the reflectance c of the mixture is

(The reflectance of the mixture c is the arithmetic mean of the two colours, $c = \frac{a+b}{2}$. If the colours a and b are mixed in equal quantities)

$$c = \frac{ma + nb}{m + n}$$

or, in other words, the reflectance of the mixture-colour is the weighted average of the components, the weights being the quantities of the components.

The same formula can be expressed in a more suitable form ~~using as weighs~~ if we use as weights instead of the absolute quantities the proportions (summing up to 1) of the components. Putting $\frac{m}{m+n} = \alpha$ and $\frac{n}{m+n} = 1 - \frac{m}{m+n} = 1 - \alpha$ the above equation takes the form

$$c = \alpha a + (1 - \alpha) b,$$

(in mezzo alla pagina)

$\alpha$ and $1 - \alpha$ being the proportions in which to components are present in the mixture.

But if Talbot's law rules both phenomena, colour mixture and colour scission, then the above equation, describing colour mixture, describes also colour scission. ~~In this case~~ Being suitable to use symbols according to the preceding example, the colour scission- or transparency equation will be used from now on in the form $p = \alpha a + (1-\alpha) t$, where p is the reflectance of the stimulus-colour, a the reflectance of the second layer, ~~xxxx~~ t the reflectance of the first and trasparent layer, and $\alpha$ and $(1-\alpha)$ are the quantities or ~~exacter~~ more exactly the pro-

(1) According to Ostwald, ~~(Mathetische Farbenlehre p. ...)~~ the law of colour mixture goes back to Newton).

(in mezzo alla pag.)

regions are transparent in each every figure; that is, the contiguous regions where the splitting phenomenon takes place are always the regions between which there is less difference in reflectance. In this case the relation $|a-b| > |p-q|$ has decided which regions (that is become the splitting regions) take the functions of p and q, and which take the functions of a and b. Therefore, also in this case, the above necessary condition has been respected.(1)

The second condition can be espressed by saying that the brightness gradient (or the fall of brightness-level) between p and q on the one hand ~~one sides~~ and between a and b on the other hand ~~othersides~~ must have the same direction. (2)

? on the one hand ... on the other ...

Nota 2 (2) The formulation can be simplified to $(p > q) \Longleftrightarrow (a > b)$ or, yet further simpler~~,~~ to $p > q$ if we define $a > b$, or, in other words, the brighter of the two shades of grey~~,~~ which form the ground, is named a~~: In this case, the necessary condition is $p > q$.~~

Also this condition may be ~~submitted to a~~ controlled. On Fig. ~~11~~ 4 where transparency is generally perceived this condition is respected, because the splitting region which is contiguous to ~~A, namely~~ to the brightest region, the P region, is clearer than the other splitting region, Q. If we reverse the brightness gradient between P and Q (Fig. 5 ~~16~~) ~~it is not possible to perceive~~ transparency is not more perceived.

(1) This is, of course, only a first and very rough check. Others checks can be done ~~using models or~~ constructing models where $|p-q|$ approaches $|a-b|$. In these cases the perception becomes ambiguous: there is alternation between transparency of the central region (like Fig. 8) and transparency of the peripheral region (like Fig. 9). In other words the law $|a-b| > |p-q|$ holds only when the difference between $|a-b|$ and $|p-q|$ reaches a certain size.

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is ~~more~~ darker than each of the regions A P Q B; in the case AD, that is when a is brighter than p, and q is brighter than b, the brightness sequence is exactly defined by $a > p > t > q > b$ and the transparent layer T is brighter than the regions Q and B, and darker than A and P.

In the case BD, which is the contrary of the case AC, the transparent layer T is the brightest. The BC combination repeats the case AD, where the transparent layer T has a midway position in the brightness sequence; only in this case b is brighter than a, and q is brighter than p, a brightness relation which had been excluded by fixing $a > b$ in order to avoid useless repetitions.

o→o

The theory ~~has been~~ can be applied also in special cases, when the different shades of grey are 3 instead of 4 (if $a=p$ or $a=q$; $a=b$ and $p=q$ being excluded because in this case $\alpha$ trespasses the field of validity); and also to the cases of more then 4 different fields of grey).

to o in?

From the preceding analysis three different cases ~~are~~ follow:

1. The transparent layer T is the darkest of the sequence (case AC).

That is, of a brighter

If $a > p$ and $b > q$, $t$ is the darkest of all. Combining conditions ~~case~~ A and C two sequences, $a > p > b > q > t$ (Fig. 12) and $a > b > p > q > t$ (Fig. 13) are generated ~~Whence it follows the necessary conditions of transparency are not~~ ~~contr~~ which are not conflicting with the necessary conditions of transparency(1)

2. ~~The colour of~~ The transparent layer T is darker than a and p and brighter than q and b (cases AD and BC(2)). The sequence obtained combining conditions ~~case~~ A and D is $a > p > t > q > b$ (Fig. 8, 10, 11), which is not conflicting with the conditions of transparency.

If $a > p$ and $q > b$, t is midway between the two pairs of colours

3. The transparent layer T is the brightest of the sequence (case BD). If $p > a$ and $q > b$, t is the brightest of all. Combining conditions ~~case~~ B and D, two sequences $t > p > a > q > b$ (Fig. 14) and $t > p > q > a > b$ (Fig. 15) are generated, which are not conflicting with the necessary conditions of transparency.

Fig 12, 13, 8, 14, 15, ~~which~~ that correspond to cases 1a, 1b, 2, 3a, 3b can avail as a first control of the above deductions.

o—o

(1) $a > p$ and $b > q$ can also be combined to a sequence $a > b > q > p$ which would conflict with the necessary condition $(a > b) \Leftrightarrow (p > q)$ and in fact does not give transparency. The combination $a > p = b > q > t$ (~~only three~~ where there are only 3 different shades of grey) does not conflict with the necessary conditions of transparency; ~~and then~~ ~~it really~~ in fact under these conditions transparency can be perceived.

(2) Case BC repeats case AD if a, instead of being the brightest, is the darkest of the four ~~areas~~ regions

(b)

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# A RESEARCH ON THE PERCEPTION OF TRANSPARENCY

In the research on Perception, important results have been reached through the study of "illusions", namely phenomena where perception fails to give correct information about the objective situation.

In the case of the perception of transparency, illusions help us to "see" the very problem. In fact, under normal conditions, where we are perceiving as transparent what is permeable by luminous radii, there seems not to be any perceptual problem. But if we put a sheet of transparent plastic, or a piece of glass on an homogeneous background, we do not perceive it as transparent, while in situations like Fig. 1, where there is no physical transparency, we perceive transparency (1); hence it is natural to ask oneself the reason for these facts.

Fig. 1

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(1) It could be objected that in this case, as in all the examples given in this paper the impression of transparency is incomplete, poor, and however different from a "true" transparency, as perceived when we look through a (coloured) glass of a window, or through the surface of the sea.

The reason for this difference is that in our pictures three-dimensionality is lacking, which is not a necessary, but a very powerful condition supporting the impression of transparency. But comparing real (= physical) transparency with apparent transparency under the same conditions of bidimensionality (that is, comparing a chess board with a film of grey transparent plastic on it, with a formally and chromatically identical model, but constructed as a mosaic by juxtaposition of 4 different shades of grey opaque paper, like Fig. 4) the impression is the same, and nobody can say where there is physical transparency and where not.

An initial explanation can be attempted in terms of cues: normal cues of transparency are lacking in the first case and present in the second (a surface seen through a film appears altered in its colour, because of absorbtion and reflection by the transparent surface). But with irregular figures, like Fig. 2 and 3, it is by no means clear why we perceive in Fig. 2 two surfaces one seen through the other, and not several juxtaposed blots as in Fig. 3.

Fig. 2 Fig. 3

In general terms, the problem of perceptual transparency can be stated as follows: under which conditions the stimulation of a group of visual receptors of the retina gives rise to the perception of two surfaces, one of which is seen through the other?

This problem has been evidenced and discussed by Helmholtz and Hering (1), and further subjected to experimental researches by Fuchs, Tudor-Hart, Heider, Koffka, Metzger, Kanizsa, Metelli.

It is easy to show that there are two orders of conditions of perceptual transparency, figural and colour-conditions: in fact it is possible to abolish perceptual transparency by altering form (Fig. 2,3) or colour (Fig. 4,5).

Fig. 4 Fig. 5

(1) H. Helmholtz - Physiologische Optik, p.407 and following.
E. Hering - Über die Theorie des simultanen Kontrastes von Helmholtz, 4. Mitteilung, Pflügers Archiv 43, 1888

The subject of this paper is a research into the colour conditions in perceptual transparency (1). As the purpose of this study was to find a quantitative law about the influence of colour on the perception of transparency, the first task has been to chose a quantitative expression of colour.

It is well known that for the definition of a colour not less than 3 numbers are needed, while for the definition of the different shades of grey, from white to black (which are named achromatic colours) only one number is needed, namely the index of reflectance or albedo, the amount of light reflected from a unit area divided by the amount of light it receives (the formula being $L = \frac{i}{I}$ , where L stands for the coefficient of reflection. or albedo, i stands for the intensity of reflected light and I for the intensity of light falling into the area). Since an absolute white reflects the whole amount of light that it receives, while the absolute black absorbs the whole light falling on it, the absolute white has reflectance 1, the absolute black reflectance 0 and the various shades of grey have coefficients of reflectance between 0 and 1.

For this reason - the achromatic colours being univocally defined by one number it appeared suitable to begin by confining my study to the field of acromatic colours, which from now on will be named simply colours.

Let us start from a simple figure (Fig. 6) where subjects normally perceive transparency. The figure has been chosen because it reproduces the situation of transparency obtained by an episcotister - a rotating wheel with open sectors -; the following considerations stand for both situations, but they are easier to follow if we start with a model where perceptual transparency is obtained by the method of juxtaposition of opaque surfaces (2).

Fig. 6

Fig. 6a

(1) This problem has been considered from a special point of view in F. Metelli - Zur Analyse der phänomenalen Durchsichtigkeitserscheinungen (Gestalt und Wirklichkeit, Festgabe für F. Weinhandl, Berlin, 1967), that is chiefly devoted to the theory and to a first analysis of the figural conditions of transparency.

(2) This method is due to W. Metzger (Gesetze des Sehens, II^ ed. 1953, pp. 127-8). In the figures of this paper phenomenal transparency is obtained by juxtaposition of opaque surfaces.

In this figure - which can be considered as a general model for perceptual transparency phenomena - we distinguish 4 different regions, with four different shades of grey; we name them A P Q B (capital) and the respective reflectances a p q b (small). The stimulation originated by the P region produces 2 different perceptual effects: we see an anterior layer T, which is transparent, and through this a second layer, the latter being of the same colour as the contiguous region A. (The same observation can be made for the region Q, but for the moment, let us confine our argument to P) (1). Therefore the perceptual phenomenon of transparency has been described as a case of perceptual scission - one of the much studied scission-phenomena where one sort of stimulation produces two effects, as for example surface-colour and illumination (2).

At this point it is natural to ask oneself what is the relation between stimulus colour and scission colours. In our figure we can easily perceive them all: the stimulus colour if we isolate the P region, and the scission colours when we perceive transparency.

A simple solution to the above problem is due to G.Heider and K. Koffka, who stated, and gave experimental proof that scission colours are such that, mixed together according to Talbot's law, reproduce the stimulus colour.

The example given by Koffka in his treatise (3) is as follows. If the stimulus colour is grey, and the conditions require that one of the scission colours be blue, then the other scission colour must be yellow. Symbolically, if Y + B = G, then G - B = Y.

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(1) It has to be stressed that every figure in this paper can be described according to the preceding scheme: there are always two non-splitting regions, A and B, which constitute the ground, and two splitting regions, P and Q, P splitting into A and T, and Q splitting into B and T. One needs only to add that most figures are serial repetitions of the sequence APQB alternated with the inverse sequence BQPA. (See for ex. Fig. 1,2,4 where two parts can be distinguished, an upper and a lower part, where the two inverse sequences are easy to identify; and Fig. 8-15, where several horizontal sequences are identifiable).

(2) In fact, in this case also, as in the colour constancy phenomenon, if we isolate the region which is represented twice (that is, in Fig. 6, the circle, which is perceived as a transparent layer and as a part of the ground seen through it) the perceptual scission disappears, and we perceive only one layer.

(3) K. Koffka - Principles of Gestalt Psychology, New York, 1935, pp. 260-264.

Heider and Koffka's theory is the starting point of this research.

It is clear that Heider and Koffka's formulation is not an algebraic formulation of the problem, because B , Y and G do not symbolize numbers. But it is possible to give an algebraic formulation to the problem if we confine ourselves to acromatic colours, using reflectance as their measure.

My line of reasoning has been the following. If according to Heider and Koffka's statement, the same law (that is Talbot's law in the special case of mixture through the colour wheel (1)) rules colour-mixture and colour scission, then it is possible to use the law of colour mixture for an algebraic description of colour-scission.

Talbot's law says that if two acromatic colours, whose reflectances are a and b are mixed in equal quantities, the reflectance of the mixture c is the aritmetic mean of the two colours, $c = \frac{a+b}{2}$. If the colours a and b are mixed in quantities m and n (m being the quantity of a and n the quantity of b) the reflectance of the mixture c is

$$c = \frac{ma + nb}{m+n}$$

or, in other words, the reflectance of the mixture-colour is the weighted average of the components, the weights being the quantities of the components.

The same formula can be expressed in a more suitable form if instead of the absolute quantities, we use as weights the proportions (summing up to 1) of the components. Putting $\frac{m}{m+n} = \alpha$ and $\frac{n}{m+n} = 1 - \frac{m}{m+n} = 1 - \alpha$ the above equation takes the form

$$c = \alpha a + (1 - \alpha)b$$

$\alpha$ and $1 - \alpha$ being the proportions in which the components are present in the mixture.

But if Talbot's law rules both phenomena, colour mixture and colour scission, then the above equation, describing colour mixture, describes also colour scission. Since it is convenient to use symbols according to the preceeding example, the colour scission equation will be used from now on in the form

$$p = \alpha a + (1 - \alpha)t$$

where p is the reflectance of the stimulus-colour, a the reflectance of the second layer, t the reflectance of the first and transparent layer, and $\alpha$ and $(1-\alpha)$ are the quantities or more exactly

(1) According to W. Ostwald ( Mathetische Farbenlehre, Leipzig 1921, p. 131) the law of colour mixture goes back to Newton.

the proportions into which the stimulus colour has been divided in the two layers (1).

But what meaning is to be given to quantities in this case? Or rather how will perceptually appear different quantities of colour when distributed to equal surfaces?

Different quantities distributed to equal surfaces can give as a result only a difference in colour density. And on the first layer a difference in colour density can appear only as a difference in transparency, while on the second layer it can reveal itself only through a difference in intensity which is correlative to the transparency of the first layer. In other words, great density on the first layer and little density on the second means little transparency of the first and little intensity or little visibility of the second layer; while less colour on the first layer means little density or great transparency, and more colour on the second layer means great intensity and great visibility.

Let us check this interpretation with the equation. What happens if $\alpha = 0$, that is, if the first layer T gets the whole colour of P ? In this case the equation reduces to $p = t$, and the second layer, having no colour at all, is not visible: we see only the first layer T, which is completely opaque. Thus $\alpha = 0$ defines the limiting case of the perception of an opaque surface on the top of another surface.

And what happens if $\alpha = 1$? In this case it is the second layer A which gets the whole colour (the solution of the equation being $p = a$) and the first layer T, having no colour, disappears completely. This means that the first layer is perfectly transparent and therefore wholly invisible.

In both these cases ($\alpha = 0$ and $\alpha = 1$) there is no colour scission and obviously no perception of transparency: colour scission takes place only in the inermediate cases, that is when $\alpha$ is between 0 and 1.

Which then is the measing of the coefficient $\alpha$, which has a maximum value 1 when the transparency is perfect, a minimum value 0 when there is no transparency at all, and has a high value when a little quantity of the scission-colour comes to the first layer, and therefore the transparency is great, and a low value if a great quantity of the scission colour comes to the first layer, and therefore the transparency is little?

The obvious inference seems to be that $\alpha$ - which measures the proportion of colour going to the second layer seen through the transparent one (the intensity or clearness with which the se

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(1) It is perhaps opportune to remember that the P region is divided into two layers, A and T, and the surface of each one is equal to the surface of P.

cond layer is perceived) - measures the degree of transparency and is therefore a coefficient of transparency. But quantitative experiments show beyond any doubt that transparency depends also on the chromatic quality of the first layer, and therefore $\alpha$ is only a factor of transparency. A second factor is phenomenal colour, that is colour measured by the logarithm of the reflectance (1).

Thus $\alpha$ is the index of phenomenal colour-scission; it acquires the function of a coefficient of transparency only if the other conditions (first of all $\underline{a}$ and $\underline{t}$ colours (2)) are held constant. And, phenomenal colour scission being a necessary condition of transparency, the latter is possible only if $\alpha$ is less then 1 and more then 0.

With this definition of $\alpha$, the above equation should not contain any unknown symbol, as $\underline{a}$ and $\underline{t}$ are the reflectance coefficients which measure the colours of the scission layers. There is, however an important difference between $\underline{a}$ and $\underline{t}$: $\underline{a}$ is the colour of the contiguous region A, and is therefore a known quantity (one of the known terms of the problem), while $\underline{t}$, together with $\alpha$, are the unknowns of the equation (3).

In fact the problem of phenomenal colour scission can be given the following formulation: starting from the knowledge of the

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(1) All other conditions being equal, the transparency of an achromatic surface is an inverse function of the whiteness (or the reflectance) of the surface. The inverse relation between reflectance and phenomenal transparency can be easily shown using episcotisters of equal opening and different shades of grey.

(2) Experimenting with an episcotister it is easy to show that if the colour $\underline{a}$ ($\underline{b}$) of the ground and the colour $\underline{t}$ of the rotating sector are kept constant, the transparency is a direct function of $\alpha$, the o p e n sector measured as a proportion of 360° ($\alpha = \frac{360 - \varphi}{360}$, $\varphi$ being the size of the r o t a t i n g sector).

(3) This sentence refers to the transparency cases experienced in everyday life or to the models discussed here like Fig. 1, 2, 4, 6 etc. In these cases we can measure the reflectances of the regions $\underline{A}$ and $\underline{P}$ ($\underline{B}$ and $\underline{Q}$), while we do not know $\alpha$ and the reflectance of the transparent layer T. In the special case of transparency obtained through an episcotister the known terms of the problem are the colour of the ground $\underline{a}$ ($\underline{b}$), the colour $\underline{t}$ of the rotating sector, and the size of the open sector $\alpha$.

colours (or reflectances) of the regions A and P, is it possible to predict the proportion $\alpha$ in which the colour p is divided among the two layers, and the colour of the transparent layer t?

The answer is of course a negative one, bacause the unknowns are two, and therefore the equation is indeterminate.

It is perhaps suitable to look more deeply into the question, because it can seem that, when the quantity p is divided into a and t, if p and a are known, t is determined.

A way of clarifying the question is to begin by putting it into very simple terms. If p is divided into equal parts, the equation simplifies to $p = \frac{a+t}{2}$ or $p = \frac{1}{2}a + \frac{1}{2}t$, and in this case once a is chosen, t is determined. If p is a point on a segment which represents the sequence of numbers from 0 to 1, which identifies the reflectances, that is the measures of achromatic colours, a and t have to be on opposite sides of and at the same distance from p. That is, p splits into two colours, each of which is half the quantity of p, and which, mixed together once again give p (the same colour in the same quantity).

But following the above line of reasoning we have renounced to one dimension of change, by fixing the proportion of splitting as 0,5 to 0,5. If we choose another splitting ratio, e.g. 0,25 to 0,75, and again consider p as a point on the 0-1 segment, a and t have also to be on opposite sides, but not at the same distance from p: the distance of t from p has to be 1/3 the distance of a from p. Is means that if p and a are fixed, the colour of t (that is, his position on the 0-1 scale of the reflectances) depends on the splitting ratio of p.

But so far we have used only one half of the A P Q B model (Fig. 7) and thus only one half of the data. In fact we can write a second equation using b and q, that is

$$q = \alpha' b + (1 - \alpha')t'$$

and if we may put $\alpha = \alpha'$ and $t = t'$, which seems to be very often, if not always, right (1), the system of two equations with two unknowns is soluble. The solutions are

$$\alpha = \frac{p-q}{a-b} \qquad t = \frac{aq - bp}{(a+q) - (b+p)}$$

(1) There are cases where the perceived transparency of P and Q is not the same. As the colour of the transparent layer T appears the same on both regions (that is $t = t'$) it seems reasonable to hypothesize $\alpha \neq \alpha'$, namely that in these cases the quantity of colour assigned to T in the splitting process is different for the part of T covering A from that covering B.

At this point it seems opportune to check the deduced formulas, that is to see if and to what extent there is a correspondence between theoretically deduced formulas and the facts.

Let us begin with the equation of the phenomenal scission index $\alpha$. The equation defines the field of transparency, because transparency is possible only for the values of $\alpha$ between 0 and 1; because as we have already seen $\alpha = 0$, (perfect opacity) and $\alpha = 1$ (perfect transparency) are the limiting cases where phenomenal scission is lacking, and therefore not to be considered among transparency phenomena. $\alpha > 1$ and $\alpha < 0$ would mean that one or the other layer would receive a negative quantity of colour, a situation which is devoid of meaning.

Therefore two necessary conditions follow from the formula

1. $|a-b| > |p-q|$ (otherwise $\alpha \not> 1$)
2. $(a > b) \Longleftrightarrow (p > q)$; $(a < b) \Longleftrightarrow (p < q)$ (otherwise $\alpha < 0$)

The first condition says that the difference between the reflectances of the regions A and B has to be greater than the difference between the reflectances of the splitting regions P and Q (1). Negatively expressed it becomes a sufficient condition: if the difference in reflectance between the splitting regions P and Q is greater than the difference between the non splitting regions A and B, there cannot be transparency.

Fig. 7 Fig. 8 Fig. 9

This condition can be easily controlled. Fig. 6 and 7 show two identical models with the only difference that in Fig. 6 the difference in reflectance between the internal semicircular regions is clearly less than the difference in reflectance between the external regions, while in Fig. 7 the contrary in true. The result is that in Fig. 6, and not in Fig. 7 transparency is perceived. It is interesting to see what happens in another sample of fi

(1) As measures of reflectances are higher for lighter than for darker shades of grey, in terms of (achromatic) colour the > symbol means "brighter than", and the < symbol "darker than".

gures (Fig. 8 and 9) which are constructed following the same principle (1). In this case both figures can be perceived as transparent, but different regions are transparent in each figure; that is, the contiguous regions where the splitting phenomenon takes place are always the regions between which there is less difference in reflectance. In this case the relation $|a-b| > |p-q|$ has decided which regions take the functions of $p$ and $q$, (that is, become the splitting regions) and which take the functions of $a$ and $b$. Therefore in this case also, the above necessary condition has been respected (2).

The second condition can be espressed by saying that the brightness gradient (or the fall of brightness-level) between $p$ and $q$ on the one hand and between a and b on the other hand must have the same direction (3).

This condition also may be checked. On Fig. 4, where transparency is generally perceived, this condition is respected because the splitting region which is contiguous to the brightest region A, the P region,is brighter than the other splitting region, Q. If we reverse the brightness gradient between P and Q (Fig. 5) transparency is no longer perceived.

Fig. 10 Fig. 11

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(1) Fig. 8-15 are alternations of the sequences B A P Q B A and A B Q P A B. The critical sequences, necessary for the arousal of perceptual transparency are, of course A P Q B and (or) B Q P A. The addition of the squares A and B at the beginning and at the end of the sequence has only the effect of rendering coercive and therefore general among subjects the perceptual organisation which gives rise to phenomenal transparency.

(2) This is, of course, only an initial and very rough check. Other checks can be done constructing models where $|p-q|$ approaches $|a-b|$. In these cases the perception becomes ambiguous: there is alternation between transparency of the central region (like Fig. 8) and transparency of the peripheral region (like Fig. 9). In other words the law $|a-b| > |p-q|$ holds only when the difference between $|a-b|$ and $|p-q|$ reaches a certain size.

(3) The formulation can be simplified to $(p > q) \Longleftrightarrow (a > b)$ or, yet further to $p > q$ if we define $a > b$, or, in other words, the brighter of the two shades of grey which form the ground, is named $a$.

Besides other, less interesting necessary conditions, it can be derived from the splitting-index equation that if the colour $\underline{t}$ of the transparent layer T, is held constant (1), when the difference between $\underline{a}$ and $\underline{b}$ is much greater than the difference between $\underline{p}$ and $\underline{q}$, there is little transparency (Fig. 10), whereas when the difference between $\underline{a}$ and $\underline{b}$ is hardly greater than the difference between $\underline{p}$ and $\underline{q}$, transparency is great (Fig. 11).

The other formula, $t = \dfrac{aq - bp}{(a+q) - (b+p)}$ is more complicated and does not offer the opportunity of deriving simple predictions. Nevertheless a way has been found to derive qualitative predictions also about the colour t of the transparent layer.

The original equation $p = \alpha a + (1-\alpha)t$ can be given the form $\alpha = \dfrac{p-t}{a-t}$. As it has already been said, in the case of transparency we have $0 < \alpha < 1$.

Let us consider first the disequation $\alpha > 0$, and therefore

$$1.\quad \frac{p-t}{a-t} > 0$$

This condition implies that numerator and denominator of the fraction are either both positive or both negative. We have therefore to take into consideration two cases.

CASE 1A

As numerator and denominator are positive, we have

$(p-t) > 0, \quad (a-t) > 0$

therefore $p > t$ and $a > t$

or

if $p > t$ then $a > t$ and vice versa

or

$$\boxed{(p > t) \Longleftrightarrow (a > t)}$$

$(p > t)$ implies and is implied by $(a > t)$

CASE 1B

As numerator and denominator are negative, we have

$(p-t) < 0, \quad (a-t) < 0$

therefore $t > p$ and $t > a$

or

if $t > p$ then $t > a$

or

$$\boxed{(t > p) \Longleftrightarrow (t > a)}$$

$(t > p)$ implies and is implied by $(t > a)$

Now let us consider the second disequation, namely $\alpha < 1$ and therefore

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(1) The necessity of keeping constant the colour of the transparent layer T depends on the formerly stressed fact that colour itself is a factor of transparency.

$$2. \quad \frac{p-t}{a-t} < 1$$

with reference to both cases A and B.

CASE 2A

As (a-t) is positive, multiplying both members of the disequation by (a-t), the direction of the disequation remains unchanged

$$\frac{p-t}{\cancel{a-t}} \cancel{(a-t)} < 1\ (a-t)$$

that is $(p-t) < (a-t)$

and therefore $\boxed{p < a}$

ASSOCIATING 1A and 2A

namely from the hypothesis that numerator and denominator are positive, that is

from $(p > t) \Longleftrightarrow (a > t)$

and $p < a$

it follows that

$$\boxed{a > p > t}$$

CASE 2B

As (a-t) is negative, multiplying both members of the disequation by (a-t), the direction of the disequation changes

$$\frac{p-t}{\cancel{a-t}} \cancel{(a-t)} > 1\ (a-t)$$

that is $(p-t) > (a-t)$

and therefore $\boxed{p > a}$

ASSOCIATING 1B and 2B

that is, from the hypothesis that numerator and denominator are negative, that is

from $(t > p) \Longleftrightarrow (t > a)$

and $p > a$

it follows that

$$\boxed{t > p > a}$$

Till now the consequences which followed from the above formula prove only what was a natural expectation, namely that when a phenomenal scission occurs, if one of the scission colours a or t is brighter than the stimulus colour p, then the other scission colour (t or a) is necessarily darker.

But so far we obtained the above inferences only about the areas A and P. Obviously, following the same line of thought, the same algebraic relations are obtained about the areas B and Q.

The conditions are, therefore, for the areas A and P

A. $a > p > t$ or B. $t > p > a$

and for the areas B and Q

C. $b > q > t$ or D. $t > q > b$

Putting together each of the alternative conditions for the areas A and P with each of the alternative conditions for the areas Q and B, the following combinations are obtained.

| AC | AD | BC | BD |
|---|---|---|---|
| $a > p > t$ | $a > p > t$ | $t > p > a$ | $t > p > a$ |
| $b > q > t$ | $t > q > b$ | $b > q > t$ | $t > q > b$ |

The meaning of this operation is that by choosing a given brightness order for the regions A P Q B, we can predict the place of $\underline{t}$, the colour of the transparent layer, on the above sequence or brightness scale.

Of course, the occurrence of phenomenal transparency is not guaranteed by these combinations, because transparency conditions were taken into account for every half figure separately and not for the figure as a whole. Therefore, if the previously inferred necessary chromatic conditions of transparency are lacking, transparency cannot be perceived. The meaning of the combined brightness sequences is, therefore, that if transparency occurs, then, from the brightness relations between the areas, the degree of brightness of the transparent layer can be predicted.

From the preceeding analysis three different cases follow

1. The transparent layer T is the darkest of the sequence (case AC). If $\underline{a} > \underline{p}$ and $\underline{b} > \underline{q}$, $\underline{t}$ is the darkest of all. Combining conditions A and C, two sequences, $\underline{a} > \underline{p} > \underline{b} > \underline{q} > \underline{t}$ (Fig. 12) and $a > b > p > q > t$ (Fig. 13) are generated which do not conflict with the necessary conditions of transparency (1).

Fig. 12

Fig. 13

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(1) $\underline{a} > \underline{p}$ and $b > q$ can also be combined to a sequence $a > b > q > p$, which would conflict with the necessary condition $(\underline{a} > \underline{b}) \Longleftrightarrow (\underline{p} > \underline{q})$ and in fact in this case we do not perceive transparency. The combination $a > (p=b) > q > t$ (where there are only 3 different shades of grey) does not conflict with the necessary conditions of transparency: in fact under these conditions transparency can be perceived.

2. The transparenct layer T is darker than $a$ and $p$ and brighter than $q$ and $b$ (cases AD and BC (1)). The sequence obtained by combining conditions A and D is $a > p > t > q > b$ (Fig.8,10,11) which does not conflict with the necessary conditions of transparency.

3. The transparent layer T is the brightest of the sequence (case BD). If $p > a$ and $q > b$, $t$ is the brightest of all. Combining conditions B and D, two sequences, $t > p > a > q > b$ (Fig. 14) and $t > p > q > a > b$ (Fig. 15) are generated, which do not conflict with the necessary conditions of transparency.

Fig. 14

Fig. 15

Fig. 12, 13, 8, 14, 15 that correspond to cases 1a, 1b, 2, 3a, 3b can be considered an initial check of the above deductions.

The theory can be applied also in special cases, when the different shades of grey are 3 instead of 4 (if $a=p$ or $a=q$; $a=b$ and $p=q$ being excluded because in this case $\alpha$ is outside the field of validity); and also to the cases of more then 4 different shades of grey.

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(1) Case BC repeats case AD if $a$ instead of being the brightest, is the darkest of the four regions.

As qualitative controls confirmed predictions inferred from transparency equations, quantitative research has been done, in order to see if, starting from the reflectancies of the A P Q B regions, calculations of $\alpha$ and t gave acceptable values.

Before summarizing results it is however necessary to reconsider the conditions of validity of the above equations and inequalities.

First of all, equations (3) and (4), have not been obtained by generalising experimental results, but deduced from an hypothesis. Therefore they consider only the reflectancies of the A, P, Q, B regions and not other conditions, like figural ones and color contrast effects.

It is easy to notice that in our simpler models (Fig. 5) as well as in the checkboard figures, figural conditions are by no means neutral (1); however, figurally neutral models can be constructed (Fig. 16). What cannot be avoided without preventing transparency is the mutual influence of colours.

However from colour contrast and figural influences only minor disturbances should be expected. The fundamental conditions of validity of the equations defining the degree of transparency and the colour of the transparent layer are, as has been stressed above, the equality of $\alpha$ and $\alpha'$, t and t'; that is, the equality of the degree of transparency and of the colour of the transparent layer in the P and Q regions.

Phenomenal transparency appears in other forms, whose conditions have not yet been studied: besides qualitatively different forms (2), which cannot be analyzed here, there are

(1) In order to avoid figure-ground reversals, conditions have been manipulated so as to give the central region "figure" character and therefore favoring phenomenal scission in this region. In Fig. 16 where every pair of contiguous regions has the same chance of acting as splitting regions, and only chromatic conditions are influencing the phenomenon, perception is often unstable and subject to sudden changes.

(2) In some models the majority of subjects describe the transparent layer as a shadow (when its colour is dark) or as a "stain of light" (when its colour is bright). There are also situations where the transparent layer is described as wet or polished - and therefore endowed with a sort of extra-reflectance.

quantitative differences in the degree of transparency between P and Q regions, which have to be considered very carefully to decide which equations are apt to describe the phenomenon. For this purpose we have to distinguish three cases: 1) balanced transparency (the transparent layer is homogeneously transparent). Only in this case do the above equations for $\alpha$ and t properly apply. 2) Unbalanced transparency (the transparent layer is more transparent either on the P or on the Q region. In this case the difference in transparency could possibly be evaluated (f. ex $\alpha = 2\alpha'$) and the system of equations in 2 unknowns solved. 3) Partial transparency: the layer is transparent on the P or the Q region only (see Fig. 17).

Fig. 16

Quantitative control of the validity of the $\alpha$ and t formulae has been accomplished in two different ways: namely a) about 70 models like Fig. 8-15 have been constructed, combining different shades of achromatic colours of known reflectance, and $\alpha$ and t values have been calculated for each model. b) experiments have been done with an apparatus like that shown in Fig. 16, where 3 of the 4 squares were changed systematically with different shades of achromatic colour of known reflectance, while in the fourth one a "colour-variator" changed continuously its achromatic colours from white to black. As a result, two classes of transparencies could be distinguished, a first one where $\alpha$ and t values were compatible, and a second one where the values were outside the realm of validity of the above indices. The two classes differed in the following ways: a) models of the type $a > p > q > b$ always gave compatible results, while in the other four brightness sequences non compatible results also appeared. b) compatible results were obtained in cases of "balanced transparency", that is where the degree of transparency at the P and the Q region was equal or similar, while uncompatible results were obtained where transparency was clearly unbalanced or partial. As the transparency equations are valid only for cases of balanced transparency, this result can be taken as an empirical confirmation of their validity.

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At this point it seems suitable to discuss the meaning of the formulations reached through the quantitative expression of the phenomenon of transparency and their related algebraic developments.

First of all, the quantitative expression of the phenomenon (equations (1) and (2)) mades clear that there are two parameters of transparency, $\alpha$ and t, the degree of transparency and the color of the transparent layer. Until the equation of color fusion had been used to describe transparency, it was not clear (for ex. in Heider and Koffka's formulation) that two variables had to be taken into account, and it could appear that, knowing the reflectancies of the stimulus color p and of one od the scission colors a, the other scission color t would already be determined.

Solving the system of two equations in two unknowns, (1) and (2), the expressions (3) and (4) were obtained, which are the laws relating the degree of transparency and the color of the transparent layer to the reflectancies of the regions determining the phenomenon.

Because of its simplicity and the opportunity it offers of inferring perceptually testable necessary conditions for transparency, only the equation of the transparency coefficient $\alpha$ has been used. But it has to be stressed that both equations (3) and (4) are necessary for determining when transparency is possible. The two equations (the solutions for $\alpha$ and for t) and the necessary conditions deduced from both, are thus functioning as a system of two filters. What one filter excludes is excluded in a definitive way (from each one of the equations sufficient conditions for n o n t r a n s p a r e n c y can be deduced); but what one filter lets go through has to pass also the screening of the other filter.

Finally it should be remembered that all preceding formulations have been deduced from a theory and not obtained empirically as a generalisation from experimental data. This fact is responsible for the extreme simplicity of the equations as expressions of conditions acting in pure form, free from undesired influences and from errors. They refer to "ideal" abstract situations not experimentally reproducible, taking into account only factors considered in the equations: the influence of figural conditions, which are always acting in concrete situations of transparency, favouring transparency more or less or making it impossible, and of phenomena of interaction among contiguous surfaces (contrast, equalisation) are not considered. The conditions of validity of the above laws are explicitly given: the $\alpha$ and t equations are valid only for the case of uniformity of transparency ($\alpha = \alpha'$) and colour ($t = t'$) of the transparent layer T.

Abnormal results ($\alpha$ and/or $t$ negative or greater than 1) indicate that for those particular values of the indipendent variables $a$, $p$, $q$, $b$ it is impossible to obtain equal transparency and equal color of the transparent layer in the P and Q regions. If in these cases transparency is perceived, it is a form of <u>unbalanced</u> transparency, for which conditions $\alpha = \alpha'$ and/or $t = t'$ are not present, and therefore equations (3) and (4) are not valid, nor are the necessary conditions inferred from them.

In the above conditions, instead of $\alpha = \alpha'$, $\alpha = c\,\alpha'$ (where c is a coefficient which can be approximately evaluated (for ex., c = 2 if the degree of transparency perceived on the P region is twice as much as that perceived on the Q region) and the same can happen for $t$. But then the system of two equations in two unknowns is no longer linear, and its solutions have not the simplicity which was their main quality.

An interesting exception is given by the case of <u>partial transparency</u> where there is transparency only on the P or on the Q region (namely, $\alpha = 0$ while $\alpha' \neq 0$; or the contrary). In this case solutions are especially simple and they allow further inferences which can be tested experimentally. It is interesting to note that almost all cases of unbalanced transparency observed till now appear to be cases of partial transparency.

Fig. 17

It has to be remembered finally that unlike the and $t$ equations and inferences drawn from them, the asymmetric relations allowing us to predict the color of the transparent layer constitute a system of propositions of a more general order, since they have been deduced without requiring the condition that = '.

A last point to be clarified concerns the parallelism between chromatic fusion and scission. Heider and Koffka's theory from which this research had its start, assert that there is a parallelism between chromatic fusion and transparency, interpreted as chromatic scission. The fact that it has been possible to use the chromatic fusion equation to describe chromatic scission, may seem to be a further confirmation of that parallelism. On the contrary, it makes evident its limits.

In chromatic fusion - we are still dealing with achromatic colours - a change in the degree of brightness of a component can be compensated for by a change of the proportion in which this component is present in the mixture; e.g., the same result can be obtained by adding to black a little quantity of white or a greater quantity of light grey. In other words, qualitative and quantitative changes of the components have the same effect, since they both modify the fusion color.

On the contrary, in chromatic scission, scission colors and the proportion of color distribution in the first (transparent) layer and the second (seen through the first) layer give rise to different phenomena, namely colour and degree of transparency). Besides, the fact that unlike colour fusion, the process of colour scission involves four contiguous regions (three in the case of partial transparency) and is defined by a system of two equations in two unknowns is another characteristic differentiating the two phenomena.

<u>Appendix</u>. Only the equations for the privileged case of balanced transparency have been given in this paper. Since unbalanced and especially partial transparency (1) are quite common, it seems suitable to develop the equations for these cases.

Partial transparency equations are very simple. If the P or the Q region is not perceived as transparent, there is no perceptual scission in this region, and all the colour belongs to the superficial layer T; that is, if the P region is not transparent we have $t = p$ (and if it is the Q region that is not transparent, $t = q$).

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(1) While in simple models partial transparency is easily noticed and described, in more complex models, like our chekher-board figures, this aspect of the phenomenon is easily neglected by subjects. In order to get this information subjects have to be asked if the A region phenomenally goes on and is visible under P, as well the B region under Q. In some cases (as, for example Fig. 8-11) subjects do not perceive any difference, or only a slight one, while in other cases (as f. ex. Fig. 13-15) a difference between A-P and B-Q is observed: for example, in Fig. 14-15 the grey region A is not perceived under the light grey region P, while the dark region B is clearly perceived under the lighter region Q. If subjects are not asked to give an analytic description, they have only an overall impression of transparency.

Case 1. Only the P region is transparent: that is $\alpha \neq 0$ $\alpha' = 0$. In this case the equation $q = \alpha' b + (1 - \alpha')t$ reduces to $q = t$.

Then, t being a known quantity, the other equation $p = \alpha a + (1 - \alpha)t$ takes the form $p = \alpha a + (1 - \alpha)q$, and having only one unknown can be solved for $\alpha$, that is

$$\alpha = \frac{p - q}{a - q}$$

Considering again the limiting conditions $\alpha < 1$, $\alpha > 0$, we see that from $\alpha < 1$ and being by definition $p > q$, we infer $a > p$ and therefore,

$\boxed{a > p > q}$ Necessary condition for partial transparency, when $q = t$.

In this case from the other limiting condition $\alpha > 0$ we cannot infer anything new. Therefore the above condition guarantees that the value of $\alpha$ is within the validity interval.

The above relation $a$ $p$ $q$ is then the necessary condition for PQ transparency; that is, for transparency in the P region and opacity in the Q region. This means that in only three of the five order-of-brightness relations studied before, namely $a > p > b > q$ and $a > b > p > q$ and $a > p > q > b$ is case 1 of partial transparency (region P transparent and Q opaque) possible. This inference can be tested.

Another point which deserves to be stressed is that the above necessary condition does not concern the B region. That means that in this case the B region can assume any color (even the same color as A) without affecting the phenomenon. This inference can also be tested.

Case 2. Only the Q region is transparent.

Here the necessary condition is $p > q > b$. In this case the color of A is undetermined, and the phenomenon is again possible only in three of the five order-of-brightness relations of the APQB regions; namely, $a > p > q > b$, $p > a > q > b$, $p > q > a > b$.

In the cases of unbalanced transparency the difference in transparency for P and Q can be evaluated (f.ex. $\alpha = 1{,}5\alpha'$) and the system of 2 equations in 2 unknowns (which in these cases are quadratic), studied.

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