a) casi in cui si ha frequente inversione

b) casi in cui si ha impressione d'ombra o di luce

Quando $\alpha > 1$ la formula non più funziona (casi a)

casi in cui α si avvicina a 1 e β diventa enorme

(altri tipi di trasparenza e quelli dell'ombra o della luce

ISTITUTO TRENTINO DI CULTURA
TRENTO
VIA G. VERDI 26 - TEL. 32.483 - 31.526

TRENTO, ...

8

$C_1 = B_1 + N_1$
$C_2 = B_2 + N_2$

Talbot

$B_1 = .90$
$N_1 = .10$
$B_2 = .70$
$N_2 = .30$

$C_f = B_f = 80$
$N_f = 20$

$$C_f = \frac{C_1 + C_2}{2} = \frac{B_1 + B_2}{2} + \frac{N_1 + N_2}{2}$$

$$C_f = \frac{m c_1 + n c_2}{m + n} = \frac{m(B_1 + N_1) + n(B_2 + N_2)}{m + n}$$

$$= \frac{m B_1 + m N_1 + n B_2 + n N_2}{m + n}$$

$\cancel{(m+n)}$

$$= \frac{m B_1 + n B_2 + m N_1 + n N_2}{m + n}$$

$$= \frac{m}{m+n} B_1 + \left(1 - \frac{m}{m+n}\right) B_2 + \frac{m}{m+n} N_1 + \left(1 - \frac{m}{m-n}\right) N_2$$

$$C_f = B_C + N_C = \alpha B_1 + (1-\alpha) B_2 + \alpha N_1 + (1-\alpha) N_2$$
$$= \alpha B_1 + B_2 - \alpha B_2 + \alpha N_1 + N_2 - \alpha N_2$$

$$B_2 + N_2 = \alpha B_2 + \alpha N_2 - \alpha B_1 - \alpha N_1$$

$$\alpha = \frac{B_2 + N_2 - C_f}{(B_2 + N_2) - (B_1 + N_1)}$$

$$C_1 = K B + (1 - K) N$$

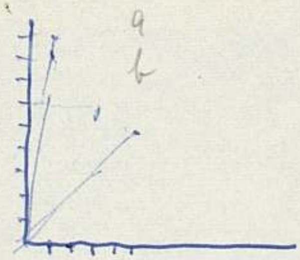
$$C_2 = \lambda B + (1 - \lambda) N$$

$$C_f = \alpha K B + (1-\alpha)\lambda B + \alpha (1 - K) N + (1-\alpha)(1-\lambda) N$$

$$= \alpha K B + \lambda B - \alpha \lambda B + \alpha N - \alpha K N + N - \lambda N - \alpha + \alpha \lambda N$$

$$\alpha = \frac{p - q}{a - b} = \frac{(p_B + p_N) - (q_B + q_N)}{(a_B + a_N) - (B_B + B_N)}$$

$p = .60$
$q\ .4$
a
b



C

1	1 B	2
↓	.9 B	
0,5	.8 B	
↓	.7 O	
1	.6 B	
	.5 BN	
	.6 N	
	.7 N	1,4
	.8 N	1,3
	.9 N	1,2
	1 N	1

E

Programmare: discussioni con Gregory; settimana con Wettinger.
Ricerca musicale
Apparato - effetto Brown

$$\frac{V_a}{V_b} = p \qquad \frac{\frac{S_a}{T_a}}{\frac{S_b}{T_b}} = p$$

$$\frac{S_a}{T_a}\,\frac{T_b}{S_b} = p \;\#\; \frac{S_a}{S_b}\cdot\frac{T_b}{T_a} = p$$

$$\frac{T_b}{T_a} = \frac{S_b}{S_a}\,p$$

$$\frac{T_a}{T_b} = \frac{S_a}{S_b}\,\frac{1}{p}$$

$\cancel{v_a = v_b \quad \frac{s_a}{t_a} = \frac{s_b}{t_b}}$

$$v_a = v_b \longrightarrow \frac{t_a}{t_b} = \frac{s_a}{s_b}$$

$$\frac{t_a}{t_b} : \frac{T_a}{T_b} = \frac{s_a}{s_b} : \frac{1}{p}\,\frac{S_a}{S_b}$$

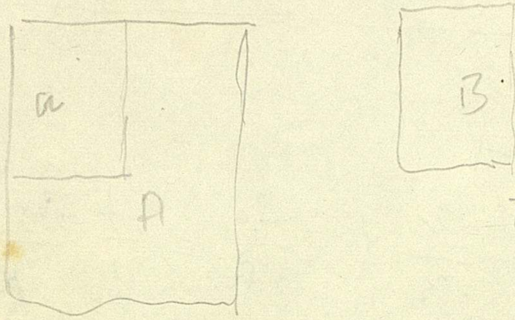
$$\frac{\frac{s_a}{s_b}}{\frac{1}{p}\frac{S_a}{S_b}}$$

$$= p\,\frac{s_a}{s_b} : \frac{S_a}{S_b}$$

$$\frac{1}{\frac{1}{p}} = p$$

se $\frac{t_a}{t_b} = 1$

$$\frac{1}{\frac{T_a}{T_b}} = \frac{p\,\frac{s_a}{s_b}}{\frac{S_a}{S_b}} \qquad \frac{T_a}{T_b} = \frac{\frac{S_a}{S_b}}{\frac{s_a}{s_b}}\,\frac{1}{p}$$



T_B

S_B

V_B

T_A

S_A

V_A

$S_a = \frac{1}{4} S_A$ $\quad s_a = \frac{1}{4} s_A$

$T_a = \frac{1}{4} T_A$ $\quad t_a = \frac{1}{4} T_A$

$V_a = V_A$ $\quad \frac{V_a}{V_B} = \frac{V_A}{V_B} = 3,80$

$v_A = v_a = v_B$

$\cancel{S_a / S_B}$

$\frac{S_a}{S_B} \frac{T_B}{T_a} = 3,80$ $\quad \frac{T_B}{T_a} = 3,80 \frac{S_B}{S_a}$

$\frac{T_a}{T_B} = \frac{S_a}{S_B} \frac{1}{3,80}$ $\quad \frac{t_a}{t_b} = \frac{s_a}{s_b}$

$p = \alpha a + (1-\alpha) t$

$p = \alpha p + (1-\alpha) t$

$p = \alpha p + t - \alpha t$

$p - \alpha p = t - \alpha t$

$p(1-\alpha) = t(1-\alpha)$

$p = t$

$p - t = \alpha(p - t)$

$\alpha = 1$ No

$p = t$

$\alpha = \frac{0}{0}$

$q = \alpha b + (1-\alpha) t$

$q = \alpha b + t - \alpha t$

~~$q - \alpha b = t(1-\alpha)$~~

$\frac{q-t}{b-t} = \alpha$

Perceptual transparency is the impression of ~~seing~~ seeing through a medium or an object. It will be shown that this phenomenon depends on special conditions; and these are not the conditions of physical transparency, namely permeability by luminous radii. As a matter of fact, physical transparency is neither a necessary nor a sufficient condition of perceptual transparency. There are situations where there is physical but not perceptual transparency, and on the contrary, situations where there is perceptual but not physical transparency, (Fig. 1, 2). I point out that this figure has been obtained by putting together pieces of cardboard, like a mosaic, not by painting

Non sufficient

N. necessari

It is ~~further~~ easy to show that there are two orders of perceptual conditions of transparency, figural and colour-conditions, because it is possibile to abolish perceptual transparency either altering form or colour. (Fig. 3, 4, 5)

The subject of this lecture is a research ~~about~~ into the colour conditions in perceptual transparency: my purpose has been to find a ~~quantitative~~ law about the influence of colour on the perception of transparency. ~~You will judge if and till where whether and to what extent I have been successful.~~

My first task has been to choose a quantitative expression of colour.

brightness
hue
saturation

The quantitative definition of a colour is a complex thing

It is well known that for the definition of a colour, not less than 3 numbers are needed, while for the definition of the different shades of grey, from white to black (which are named achromatic colours) only one number is needed, namely the index of reflectance or albedo, the amount of light reflected from a unit area divided by the amount of light it receives (the formula being $L = \frac{i}{I}$, where L stands for the coefficient of reflection or albedo, i (small) stands for the intensity of reflected light and I (capital) for the intensity of light falling into the area). ~~As~~ Since an absolute white reflects the whole amount of light that it receives, while the absolute black absorbs the whole light falling on it, the absolute white has albedo ~~reflectance~~ 1, the absolute black, albedo ~~reflectance~~ 0, and the various shades of grey have albedo ~~coefficients of reflectance~~ between 0 and 1.

For this reason - the achromatic colours being univocally defined by one number, the albedo ~~coefficient of reflectance~~ - I found it suitable to begin by confining my study to the field of acromatic colours, which from now on will be named simply colours.

Let us start from a simple figure (Fig. 6) ~~where~~ subjects ~~normally~~ in generally perceive transparency. The figure has been chosen because it reproduces the situation of transparency obtained by an episcotister - a coloured

(Lavagna)

rotating in front of a ground

~~rotating~~ wheel with open sectors -; the following considerations stand for both situations, but they are easier to follow if we start with a model where transparency is obtained with Metzger's method of juxtapposition of opague opaque surfaces. I point out that (This figure has been obtained by putting together pieces of cardboard) juxtaposing ~~as in a mosaic~~

In this figure - which can be considered as a general model for perceptual transparency phenomena - we distinguish 4 different regions, with four different shades of grey; we name ~~them~~ The regions A P Q B (capital) and the respective reflectances albedos a p q b ~~(small)~~ (small).

$P \to \begin{smallmatrix} T \\ A \end{smallmatrix}$

$P(r) \to \begin{smallmatrix} T(t) \\ A(a) \end{smallmatrix}$

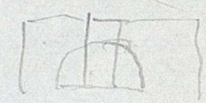
The stimulation originated by the P region produces 2 different perceptual effects: we see an anterior ~~layer T, which is transparent,~~ Transparent layer, which we therefore name T and through this one, a second layer the latter being of the same colour as the contiguous region A. (The same observation can be made for the region Q, but for the moment, let us confine our argument to P.). Therefore the perceptual phenomenon of transparency has been described as a case of perceptual scission - one of the much studied phenomena where one sort ~~in~~ of stimulation produces two effects, as ~~for example~~ in the well known example of surface colour and illumination.

(Lavagna)

At this point ~~is~~ it is natural to ask oneself ~~which~~ What is the precise relation between stimulus colour and scission colours. In our figure we can easily perceive them all: the stimulus colour if fare uno schema rigido

we isolate the P region, and the scission colours when we percei ve transparency.

It is well known that a simple solution to the above problem is due to G. Heider and K. Koffka, which stated, and gave ~~an~~ experimental proof that scission colours are such, that, mixted together ~~according to Talbot's law~~, they once again reproduce ~~, again,~~ the stimulus colour.



hole

The example given by Koffka in his treatise is as follows. If the stimulus colour is grey, and the conditions require that one of the scission colours be blue, then the other scission colour must be yellow. Symbolically, if Y + B = G, grey then grey G - B = Y.

Heider and Koffka's theory is the starting point of my re search.

It is perhaps suitable to notice that Heider and Koffka's formulation is not an algebraic formulation of the problem, because B, Y and G ~~dont~~ do not symbolize numbers. But it is possible to give ~~an~~ a tentative algebraic formulation to the problem if we confine ourselves to acromatic colours, which can be each of them univocally defined by the measure of ~~its~~ their albedo reflectance.

My line of reasoning has been the following. If ~~according to Heider and Koffka's statement,~~ the same law (~~that is Talbot's~~ in the special case of the colour wheel

It is perhaps suitable to give an example.
If we mix together two colours, a and b, whose reflectances are .10 and .40, and the mixture is made by $\frac{1}{5}$ of a and 4/5 of b, then the reflectance of the resulting colour is
$(\frac{1}{5})(.10) + \frac{4}{5}(.40) = .34$

If a has an albedo of .10 and b of .40, then we can calculate the albido of the mixture which is
$(.20)(.10) + (.80)(.40) = .34$

~~which, as I read in a book by Ostwald,~~ is due to Newton)

~~law)~~ rules colour-mixture and colour scission, then it is possible to use ~~Talbot's law~~ it for an algebraic description of colour-scission. The law is, in the case of the mixture obtained through a colour wheel

Talbot's law It says that if two acromatic colours, whose ~~re-flectances~~ albedos are ~~t~~ a and ~~a~~ b are mixted in ~~quantities m and n~~ proportions α and $(1-\alpha)$ ~~(m being the quantity of t, and n the quantity of a)~~ the reflectance of the mixture ~~is~~ c, is given by the equation

lavagna

$$\alpha a + (1-\alpha) b = c$$

$$\cancel{p = \frac{ma + nb}{m + n}}$$

or, in other words, the reflectance of the mixture-colour is the weighted average of the components, the weights being the quantities of the components.

~~The formula can be expressed in a more suitable form using as weighs~~ If instead of the absolute quantities we use as weights the proportions (summing up to 1) of the components, the formula can be expressed in the more suitable form

~~$p = \alpha a + (1 - \alpha)t$, α and $1 - \alpha$ being the proportions in which to components are present in the mixture.~~

But if Talbot's law rules both phenomena, colour mixture and colour scission, then the above equation, describing colour mixture, describes also colour scission. Using the known symbols, the equation becomes $p = \alpha a + (1-\alpha) t$, where ~~In this case~~ p is ~~the reflectance of~~ the stimulus-colour, a the ~~reflectance~~ colour of the second layer, ~~thxx~~ t the ~~reflectance~~ colour of the first and trasparent layer, and α and $(1-\alpha)$ are the quantities or ~~exacter~~ more exactly the pro-

~~positions~~ proportions into which the stimulus colour has been divided into the two layers.*

But ~~which~~ What meaning has to be given to quantities in this case? Or ~~better~~ rather how ~~express themselves~~ can appear perceptually different quantities of colour distributed to equal surfaces? (Because, as ~~it is perhaps opportune to point out, that the two layers~~, A and T have equal surface, that is, the same surface as the P region)

~~The right answer seems to be that~~ It seems to me that Different quantities distributed to equal surfaces can give as a result only a difference in colour density. And on the first layer a difference in colour density can appear only as a difference in transparency, while on the second layer it can reveal itself only through a difference in intensity which is correlative to the transparency of the first layer. In other words, great density on the first layer and little density on the second means little transparency of the first and little intensity or little visibility of the second layer, while less colour on the first layer means little density or great transparency, and more colour on the second layer means ~~great density and~~ great intensity and great visibility.

Let us ~~control~~ check this interpretation ~~on~~ with the equation. What happens if $\alpha = 0$, that is, if the first layer, that is the would be transparent layer, gets the whole colour of p? In this case $\underline{p} = \underline{t}$, and $\underline{a}$, having no colour at all is not visible, ~~this~~ thus we see only the first layer $\underline{t}$, which is completely opaque.

lavagna

~~At this point~~ * It is perhaps opportune to remember that, the colour of the P region ~~being~~ is divided into ~~the~~ two layers, A and T, the surface of each one being equal to the surface of P.

10

And what happens if $\alpha = 1$? In this case it is the second layer which gets the whole colour (the solution of the equation being $p = a$) and the first layer disappears completely. This means that the first layer is perfectly ~~xxaspx~~ transparent and therefore wholly invisible.

In both these cases ($\alpha = 0$ and $\alpha = 1$) there is no colour scission: colour scission begins only in the intermediate cases, ~~when~~ that is when, α is between 0 and 1.

Which ~~is~~ then is the meaning of the coefficient α, which has a maximum value 1 when the transparency is perfect, a minimum value 0 when there is no transparency at all, and ~~ga~~ has a high value when a little quantity of the scission-colour comes to the first layer, and therefore the transparency is great, and a low value if a great quantity of the scission colour comes to the first layer, and therefore the transparency is little?

The obvious inference seems to be that α measures the degree of transparency and is therefore a coefficient of transparency; and I must confess that ~~in the former drafts of my research-report~~ at first I yielded to this temptation. But quantitative experiments show beyond every doubt that transparency depends also on the chromatic quality of the first layer, and therefore α is only a factor of trahsparency. The second factor is phenomenal colour, that is colour measured by the logaritm of the albedo reflectance.

Thus α is ~~a coefficient of phenomenal~~ the index of perceptual colour-scission; it acqui res the function of a coefficient of transparency only if the colour of the transparent layer T ~~p~~ is held constant. And, phenomenal colour scission being a necessary condition of transparency, the latter transparency is possible only if α is less then 1 and more then 0.

With the definition of α, the above equation should not contain any unknown symbol, because as _a_ and _t_ are the ~~reflectance coefficients which measure the~~ colours of the scission layers. There is, however an important difference between _a_ and _t_ : _a_ is the colour of the contiguous region A, and is therefore a known quantity (one of the known terms of the problem), while _t_, together with α , are the unknowns of the equation.

In fact the problem of phenomenal colour scission can be given the following formulation: starting from the knowledge of the colours (or reflectances) of the regions A and P is it possible to predict the proportion α in which the colour _p_ is divided among the two layers, and the colour of the transparent layer _t_?

The answer is of course a negative one, because the unknowns are two, and therefore the equation is indeterminate.

It is perhaps interesting to look ~~deefer~~ more deeply into the question, because it seems that, ~~being~~ when the quantity p is divided into a and t, if p and a are known, t ~~be~~ is determined.

A way of clarifying the question is to begin~~ning~~ ~~with~~ by putting it into very simple terms. If p is divided into equal parts, ~~The~~ the equation simplifies to $p = \frac{a+t}{2}$ or $p = \frac{1}{2}a + \frac{1}{2}t$, and in this case once a is chosen, t is determined. If p is a point on a segment which represents the sequence of numbers from 0 to 1, a and t have to be ~~at the~~ on apposite sides of and at the same distance ~~of~~ from p. That is, p splits into two colours, each of which is half the quantity of p, and which, mixed together once again give ~~again~~ p (the same colour in the same quantity).

But following the above line of reasoning we have renounced ~~to~~ one dimension of change, ~~x~~ by fixing the proportion of splitting as 0,5 to 0,5. If we choose another splitting ratio~~n~~, f. ex. 0,25 to 0,75, and consider again p as a point ~~of~~ on the 0-1 segment, a and t have also to be ~~also~~ at ~~the~~ apposite sides, but not at the same distan ce from p: the distance of t from p has to be 1/3 the distance of a from p. ~~That~~ This means that if p and a are fixed, the colour of t depends on the splitting ratio of p; and what is more interesting, there is a compensation between the splitting ratio and the colour of the tran sparent layer]

But ~~as till now~~ so far we have used only one half of the A P Q B model, and ~~theis~~ thus only one half of the data. ~~As matter of~~ In fact we can now write a second equation using b and q, that is

$$q = \alpha' b + (1-\alpha')t'$$

and if we may put $\alpha = \alpha'$ and $t = t'$, ~~what~~ which is not always, but often right, the system of two equations with two unknowns is soluble. The solutions are

$$\alpha = \frac{p - q}{a - b} \qquad t = \frac{aq - bp}{(a+q) - (b+p)}$$

At this point it seems ~~suitable~~ opportune to ~~submit~~ check the deduced formulas ~~to a control~~, that is to see if there is a correspondence between theoretically deduced formulas and the facts.

This is easy to do [Let us begin] with the equation of the phenomenal scission index α. The equation defines the field of transparency, because transparency is possible only for the values of α between 0 and 1; ~~because~~ as we have already ~~saw~~ seen $\alpha = 0$, (perfect opacity) and $\alpha = 1$ (perfect transparency) are the limiting cases where phenomenal scission is lacking, and therefore not to be considered among transparency phenomena. $\alpha > 1$ and $\alpha < 0$ would mean that one or the other layer would receive a negative quantity of colour, ~~what~~ a situation which is devoided of

higher
lower

meaning.

Therefore two necessary conditions follow from the formula

a) $|a-b| > |p-q|$ (otherwise $\alpha \gg 1$)

b) $(a>b) \Longleftrightarrow (p>q)$, $(a<b) \Longleftrightarrow (p<q)$ (otherwise $\alpha < 0$) (double implication)

The first condition says that the difference between the reflectances of the regions A and B has to be greater than the difference between the reflectances of the splitting regions P and Q. Negatively expressed it becomes a sufficient condition: if the difference between the splitting regions P and Q is greater than the difference between the non splitting regions A and B, there cannot be transparency.

This condition can be easily ~~controlled~~ checked. Fig. 7 and 8 show two identical models with the only difference, that in Fig. 7 the difference in reflectance between the internal semicircular regions is clearly less than the difference in reflectance between the external regions, while in Fig. 8 the contrary is true. The result is that in Fig. 7 and not in Fig. 8 transparency is perceived. It is interesting to see what happens in another ~~xouple~~ sample of figures (9, 10) which are constructed following the same principles. In this xxx case both figures can be percieved as transparent, but different regions are transparent in ~~every~~ each figure; that is, the contiguous regions where the splitting phenomenon takes place are always the regions between which there is less difference in reflectance. In this case the relation $|a-b| > |p-q|$ has decided which regions take the functions of p and q, and which the functions of a and b. Therefore, ~~also~~ in this case also the above necessary condition has been respected.

The second condition can be espressed by saying that the brightness gradient (or the fall of brightness-level) between p and q at the one hand ~~one sides~~ and between a and b on the other hand ~~othersides~~ must have the same direction. The formulation can be simplified to $(p > q) \Leftrightarrow (a > b)$ ~~or, yet simpler~~ ~~further~~, ~~to p > q~~ if we define $a > b$, or, in other words, the brighter of the two shades of grey, which form the ground is named a. ~~In this case, the necessary condition is p > q.~~

~~Also~~ This condition also may be ~~submitted to a control~~ checked. On Fig. 11 where transparency is generally perceived this condition is respected, because the splitting region which is contiguous to A (namely to the brightest region), the P region, is ~~clearer~~ brighter than the other splitting region, Q. If we reverse the brightness gradient between P and Q (Fig. 12) it is not possible to perceive transparency.

* It has also to be pointed out that this formula requires a ratio scale for the measurements of colours, while the other one can be used in a way that requires only an ordinal scale

* There is no time to give ~~you~~ the algebraic deduction. The results are that if the brightness relation is

~~Kxas~~ $a > p > b > q$ the inference is that ~~t is the~~ the transparent layer t is the darkest

or

$a > b > p > q$

if $a > p > q > b$, then ~~colo~~ t is medium between p and q

if $p > a > q > b$ t is the brightest

or

$p > q > a > b$

I have to ~~add~~ add, that there are also difficulties, which have not yet been solved. But it gave me a special pleasure to be able to work this way, namely ~~infer, being able to infer~~ starting from a pure theory for making inferences and controlling them.

Besides other, less interesting necessary conditions ~~it deri~~ We can deduce ~~ves~~ from the splitting-index equation that if the colour of the transparent layer T_x is held constant, ~~xxxxx~~ when the difference between a and b is much greater than the difference between p and q, there is little transparency, ~~while~~ whereas when the difference between a and b is hardly greater than the difference between p and q, transparency is great. Also this prediction may be controlled (Fig. 13 and 14). The other formula, $t = \frac{aq - bp}{(a+q)-(b+p)}$ is more complicated and does cumbersome not offer the opportunity of deriving simple predictions. * ~~For a~~ To check this, ~~control it seams necessary to~~ one must have recourse to measurements, as we are doing at present ~~at~~ in our Laboratory. Nevertheless ~~a~~ an algebraic way has been found to ~~derive~~ deduce qualitative ~~praxix~~ predictions about the colour of the transparent layer, starting from the original equation **

The original equation $p = \alpha a + (1-\alpha)t$ can be given the form $\alpha = \frac{p-t}{a-t}$. As it has already been said, in the case of transparency α is $>$ 0 and $<$ 1.

Let us consider first the disequation $\alpha > 0$.

Therefore wa can write

$$1. \quad \frac{p-t}{a-t} > 0$$

This condition implies that numerator and denominator of the fraction are either both positive or both negative. We have therefore to take into consideration two cases.

Case 1 A

As numerator and denominator are positive, we have

$(p-t) > 0\ ,\quad (a-t) > 0$

therefore $p > t$ and $a > t$

or

if $p > t$ then $a > t$ and viceversa

or

$(p > t) \iff (a > t)$

(double implication)

Case 1 B

As numerator and denominator are negative, we have

$(p-t) < 0,\quad (a-t) < 0$

therefore $t > p$ and $t > a$

or

if $t > p$ then $t > a$

or

$(t > p) \iff (t > a)$

(implies and is implied by)

Now let us consider the second disequation, namely $\alpha < 1$ and therefore

$$2. \quad \frac{p-t}{a-t} < 1$$

with ~~With~~ reference to both ~~axx~~ cases A and B.

Case 2 A

As $(a-t)$ is positive, multiplying both members of the disequation by $(a-t)$, the direction of the disequation remains unchanged

$$\frac{p-t}{\cancel{a-t}}\,\cancel{(a-t)} < 1\,(a-t)$$

that is $(p-t) < (a-t)$

and therefore $\boxed{p < a}$

Case 2 B

As $(a-t)$ is negative, multiplying both members of the disequation by $(a-t)$, the direction of the disequation changes

$$\frac{p-t}{\cancel{a-t}}\,\cancel{(a-t)} > 1\,(a-t)$$

that is $(p-t) > (a-t)$

and therefore $\boxed{p > a}$

From 1A and 2A

(namely from the hypothesis that numerator and denominator are positive) that is

from 1A $(p > t) \Longleftrightarrow (a > t)$

and 2A $a > p$

it follows That

$\boxed{a > p > t}$

From 1N and 2B

(that is from the hypothesis that numerator and denominator are negative) that is

from 1A $(t > p) \Longleftrightarrow (t > a)$

and 1B $p > a$

it follows That

$\boxed{t > p > a}$

Till now the consequences which followed from the above formula prove only what was ~~a natural exspectancy~~ an obvious exspectation, namely that when a phenomenal scission occurs, if one of the scission colours a or t is brighter than the stimulus colour p, then the other scission colours (t or a) is necessarily darker.

But so far we obtained the above inferences only about the areas A and P. Obviously, following the same line of thought, the same algebraic relations are obtained about the areas B and Q.

The conditions are, therefore, for the areas A and P

A. $a > p > t$ or B. $t > p > a$

and for the areas B and Q

C. $b > q > t$ or D. $t > q > b$

Putting together each of the alternative conditions for the areas A and P with each of the alternative conditions for the areas Q and B, following combinations are obtained.

AC	AD	BC	BD
a > p > t	a > p > t	t > p > a	t > p > a
b > q > t	t > q > b	b > q > t	t > q > b

The meaning of this operation is that by choosing a given brightness order for the regions A P Q B, we can predict the place of t, the colour of the transparent layer on the above sequence, or brightness scale.

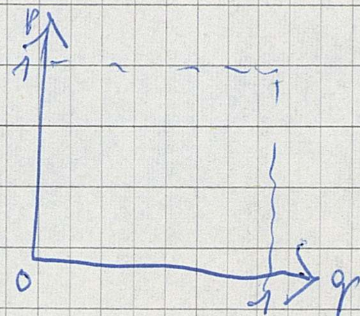
Of course, the occurrence of phenomenal transparency is not guaranteed by these combinations, because transparency conditions were taken into account for every half figure separately and not for the figure as a whole. Therefore, if the previously inferred necessary chromatic conditions of transparency are lacking, transparency cannot be perceived. The meaning of the combined brightness sequences is, therefore, that if transparency occurs, then, from the brightness relations between the areas, the brightness degree of the transparent layer can be predicted.

Thus, in the case AC, that is, if a is brighter than p, and b is ~~highter~~ brighter than q, the prediction is that the transparent layer T

The phenomenon depends on 4 variables, a, p, q, b, which are mutually independent, and whose vailues can vary between 0 and 1.

In order to symplyfie the ~~system~~ system's description ~~it is possi~~ we can fix arbitrarily the vailues of a and b (which are privileged variables, which ~~build~~ constitute the ground); so we can confine the analysis to the p and q variables.

The variation field can then be given the following geometric representation



The fundamental equations $\alpha =$ $\quad$ $p =$ $\quad$ may then be considered functions of the variables p and q, while a and b are known ~~constants and therefore arbitrarily changeable constan~~ constants and α and t parametric, and therefore arbitrarily variable constants.

~~Log.~~ Because of logical and physical reasons α and t are defined only between 0 and 1.

It is important to calculate the vailues of the variables (and represent geometrically) ~~under~~ at the conditions $\alpha = 0$, $\alpha = 1$, $t = 0$, $t = 1$

namely

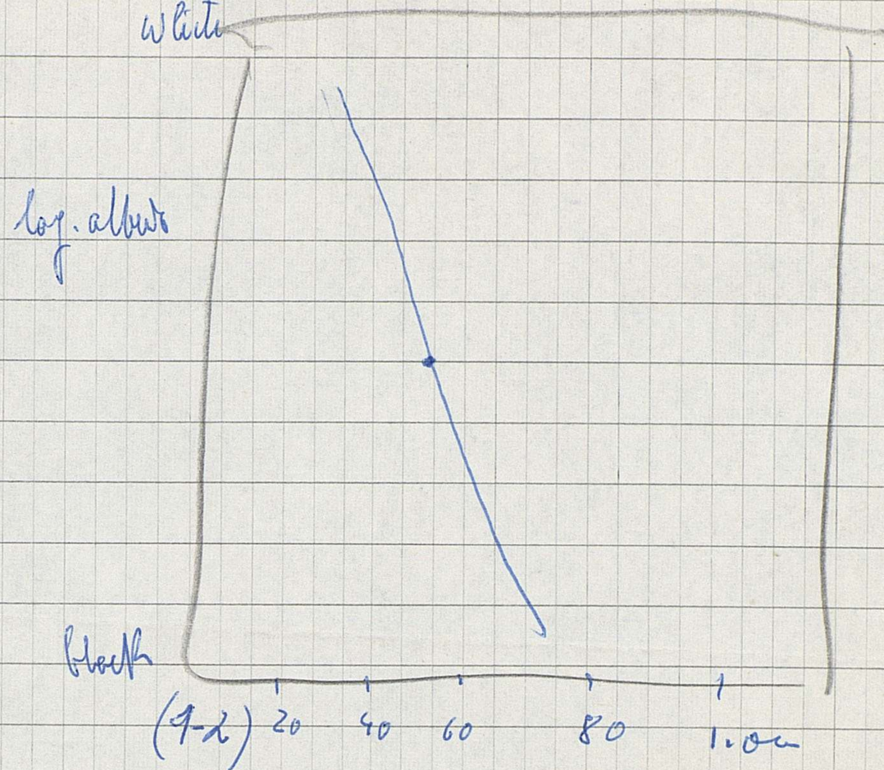
$\alpha = 0 \quad p = q \qquad\qquad t = 0 \quad p = \frac{a}{b} q$

$\alpha = 1 \quad p = q + (a-b) \qquad\qquad t = 1 \quad p = \frac{a-1}{b-1} q - \frac{a-b}{b-1}$

is ~~more~~ darker than each of the regions A P Q B; in the case AD, that is when a is brighter than p, and q is brighter than b, the brightness sequence is exactly defined by $a > p > t > q > b$ and the transparent layer T is brighter than the regions Q and B, and darker than A and P.

In the case BD, which is the contrary of the case AC, the transparent layer T is the brightest. The BC combination repeats the case AD, where the transparent layer T has a midway position in the brightness sequence; only in this case b is brighter than a, and q is brighter than p, a brightness relation which had been excluded by fixing $a > b$ in order to avoid useless repetitions.

The theory has been applied also to special cases, when the different shades of grey are 3 instead of 4 (if $a=p$ or $a=q$; $a=b$ and $p=q$ being excluded because in this case α trespasses the field of validity); and also to the cases of more then 4 different felds of grey).



1 : 5 = 0,20

Of course, ~~through this or~~ the occurrence of phenomenal transparency is not guaranteed by these combinations, because transparency conditions were taken into account for every half figure separately and not for the figure as a whole. Therefore, if the ~~above~~ previously inferred necessary chromatic conditions of transparency are ~~not present~~ lacking, transparency cannot be perceived. ~~S~~ The meaning of the combined brightness sequences therefore is, that if transparency occurs, then, from the brightness relations between the areas, the brightness degree of the transparent layer can be ~~predicted~~ predicted.

Thus, in the case AC, that is, if $\underline{a}$ is brighter than $\underline{p}$, and b is brighter than q, the prediction is that ~~T~~ the transparent ~~is not~~ to layer T is more darker than ~~all the~~ each of the regions APQB; in the case AD, that is when $\underline{a}$ is brighter than $\underline{p}$, and $\underline{q}$ is brighter than $\underline{b}$, the brightness sequence is exactly defined by $a > p > t > q > b$ and the transparent layer T is brighter than the regions Q and B, and darker than A and P. ~~To~~ In the case BD, which is the contrary of the case AC, the transparent layer T is the brightest. The ~~co~~ BC combination ~~only~~ repeats the case AD, where ~~a which~~ the transparent layer T has a midway position in the brightness sequence; only in this case $\underline{b}$ is brighter than $\underline{a}$, and q is brighter than $\underline{p}$, a brightness relation which had been excluded fixing $a > b$ in order to avoid ~~meaningless~~ useless repetitions.

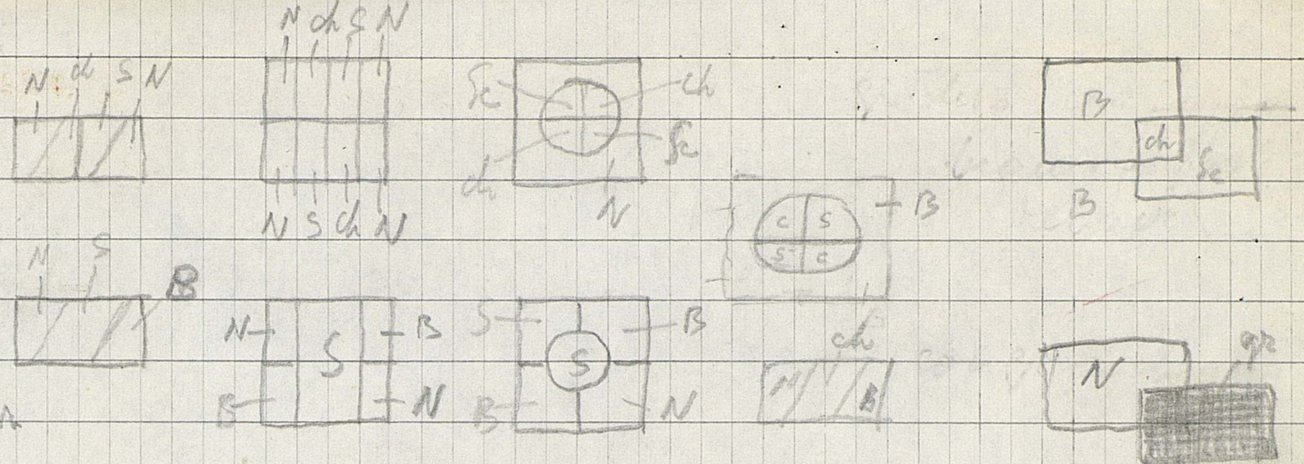
The theory ~~can~~ has been applied also on special cases, when the different shades of grey are 3 instead of 4 (if $a = p$ or $a = q$; $a = b$ and $p = q$ being excluded because in this case it trespasses the field of validity); and also to the cases of more then 4 different fields of gray).

$A = B$

esclusa la trasformaz

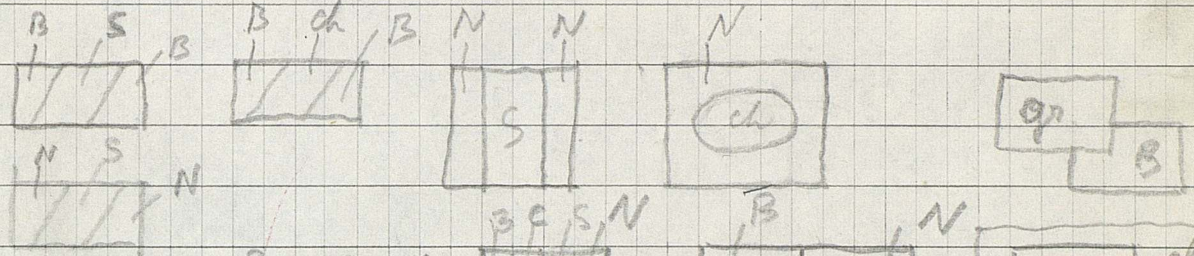
$P = Q$

esclusa la trasformaz



$A = B \,.\, P = Q$

esclusa la trasformaz



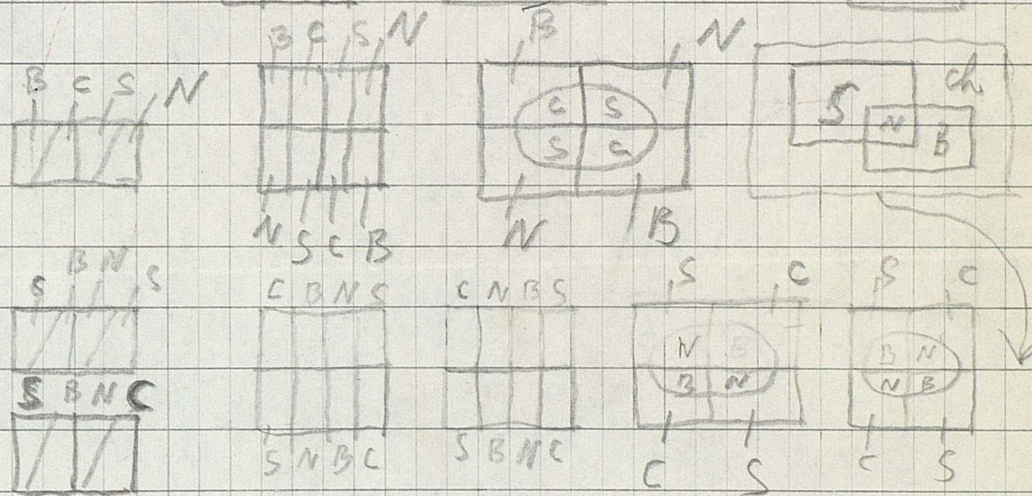
$|A - B| > |P - Q|$

$A > B \supset P > Q$

consentita la trasformaz

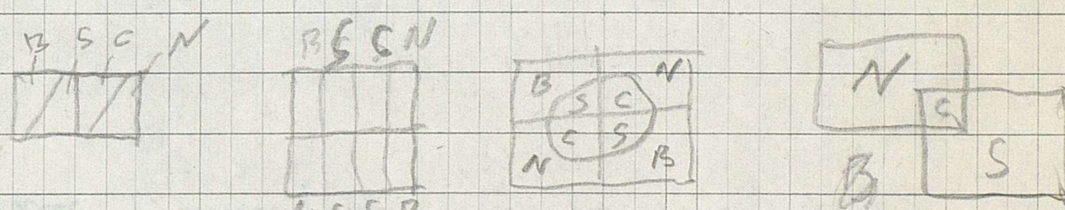
$|A - B| < |P - Q|$

esclusa la trasformaz

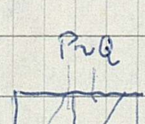


$A > B \,.\, P < Q$

esclusa la trasformaza



Trasp. massima



A bianca B nera

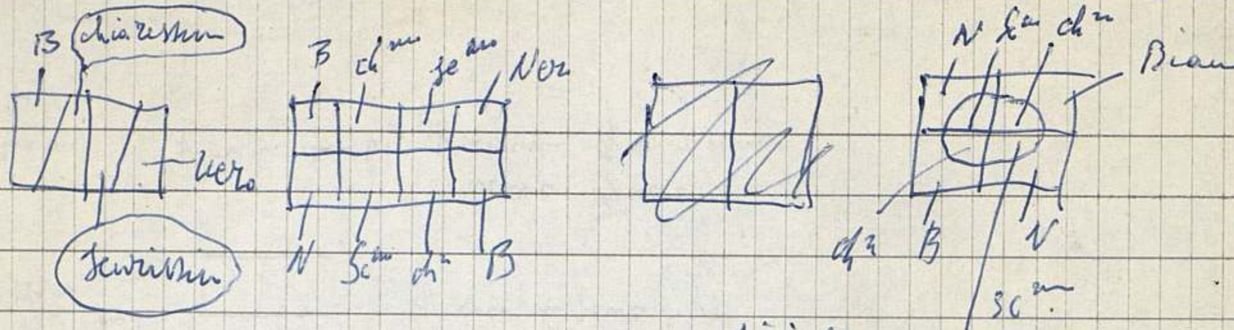
Q poco più scuro di P

Trasp. minima $p = a \,.\, q = b$

A bianca B nera

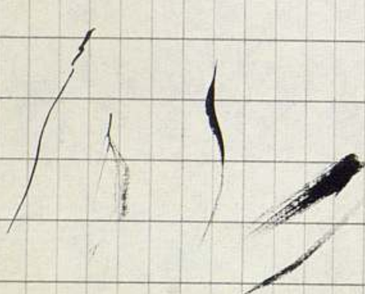
p chiarissima q scurissima

fare



simili

fare gli testi, una Bianco, poco chiar, poco scur, ner



$a = .7$
$b = .2$
$p = .6$

8) $r = .6$ $q = .14$ $\alpha = \frac{.46}{.5} = .92$ $t = \frac{.7(.14) - .12}{.5 - (.46)} = \frac{.038}{.04} = .95$ $\frac{.022}{.04}$

.098

9) $p = .6$ $q = .12$ $\alpha = \frac{.48}{.5} = .96$ $t = \frac{.7(.12) - .06}{.5 - (.48)} = \frac{.024}{.020} = 1.2$ neg.

.84

10) $p = .6$ $q = .13$ $\alpha = \frac{.47}{.5} = .94$ $t = \frac{.7(.13) - .06}{.5 - (.47)} \; \frac{.031}{.030} = 1.03$ neg.

$\frac{.056}{.060}$

11) $p = .6$ $q = .08$ $\alpha = \frac{.52}{.50} = 1.04$ $t = \frac{.7(.08) - .06}{.5 - (.52)} = \frac{-.004}{-.02}$

$p = .6$ $q = .1$ $\alpha = \frac{.5}{.5} = 1.00$ $t = \frac{.7(.1) - .12}{.5 - .5} \longrightarrow$ negativo

Invernendo

$a = 7$
$b = 2$
$p = 6$
$q = 1,4$ $\alpha = \frac{4,6}{5} = .92$ $t = \frac{7(1,4) - 6(2)}{5 - 4,6} = \frac{9,8 - 12}{0,4} = -\frac{2,2}{0,4}$

$q = 2$ $\alpha = \frac{4}{5} = .8$ $t = \frac{7(2) - 6(2)}{5 - 4} = \frac{2}{1} = 2$ [.2]

$q = 1,8$ $\alpha = \frac{4,2}{5} = .84$ $t = \frac{7(1,8) - 12}{5 - 4,2} = \frac{0,6}{0,8} = .75$ [.075]

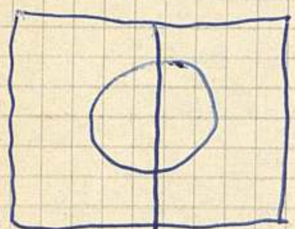
$q = 1,7$ $\alpha = \frac{4,3}{5} = .86$ $t = \frac{7(1,7) - 12}{5 - 4,3} = 0,1$ negativo

1 È possibile ottenere la seguente

$P > A > Q > B$

$P > Q > A > B$ ecc.

con l'episcotista?



h' → Conoscendo t e α è riproducibile

$a + p - r - b$

A = bianco $= .7$

B = nero $= .2$

$$\alpha = \frac{r-q}{a-b} = \frac{r-q}{.5} \qquad t = \frac{aq - pb}{(a-b)-(r-q)} = \frac{.7q - .2p}{.5-(r-q)}$$

1) $p = q = .6$ $\qquad \alpha = 0 \qquad t = \frac{(.7)(.6) - (.2)(.6)}{.5} = \frac{.42 - .12}{.5}$

? $p = \alpha a + (1-\alpha)t$, $\alpha = 0$ $\qquad \frac{.30}{.5} = .6$

$\underline{p = t} \quad \underline{q = t}$

2) $p = .6$, $q = .5$ $\qquad \alpha = \frac{0,1}{0,5} = .2 \qquad t = \frac{.7(.5) - .2(.6)}{.5 - (.1)} = \frac{.35 - .12}{.4} = \frac{.23}{.40}$

$t = \boxed{.575}$

3) $p = .6$, $q = .4$ $\qquad \alpha = \frac{0,2}{0,5} = .4 \qquad t = \frac{.7(.4) - .12}{.5 - (.2)} = \frac{.16}{.30} = .534$

4) $p = .6$, $q = .3$ $\qquad \alpha = \frac{.3}{.5} = .6 \qquad t = \frac{.21 - (.12)}{.5 - (.3)} = \frac{.09}{.20} = .45$

5) $p = .6$, $q = .2$ $\qquad \alpha = \frac{.4}{.5} = .8 \qquad t = \frac{.14 - (.12)}{.5 - (.4)} = \frac{.02}{.1} = .20$

6) $p = .6$, $q = .18$ $\qquad \alpha = \frac{.42}{.5} = .84 \qquad t = \frac{.7(.18) - (.12)}{.5 - (.42)} = \frac{.006}{.080} = .07$

7) $p = .6$, $q = .16$ $\qquad \alpha = \frac{.44}{.5} = .88 \qquad t = \frac{.7(.16) - (.120)}{.50 - (.44)} = \frac{-.008}{.060} = -.13$

$$\frac{aq - bp}{(a+q)-(b+p)}$$

$$\cancel{\frac{aq}{a+q} = \frac{bp}{b+p}}$$

$$\alpha = \frac{p-q}{a-b}$$

$\frac{p-q}{a-b}$ $(a+q) = (b+p)$ $p = a-b+q$

p q
3 2

6 4
a b

$aq = bp$ $\frac{aq}{b} = a-b+q$

$aq = b(a-b+q)$ $aq = ab - b^2 + bq$

a b p q
8 6 3 1

8 6 4 3 $\frac{8}{4}$

a a p q
8 6 3 2
p q a b
$\frac{8}{6}$ $\frac{4}{3}$ 1,3 1,5

8 6 4 2

× +

	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	0/0	0/1	0/2	0/3	0/4	0/5	0/6	0/7	0/8	0/9	0/10
.1	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/10	10/11
.2	0/2	2/3	4/4	6/5	8/6	10/7	12/8	14/9	16/10	18/11	20/12
.3	0/3	3/4	6/5	9/6	12/7	15/8	18/9	21/10	24/11	27/12	30/13
.4	0/4	4/5	8/6	12/7	16/8	20/9	24/10	28/11	32/12	36/13	40/14
.5	0/5	5/6	10/7	15/8	20/9	25/10	30/11	35/12	40/13	45/14	50/15
.6	0/6	6/7	12/8	18/9	24/10	30/11	36/12	42/13	48/14	54/15	60/16
.7	0/7	7/8	14/9	21/10	28/11	35/12	42/13	49/14	56/15	63/16	70/17
.8	0/8	8/9	16/10	24/11	32/12	40/13	48/14	56/15	64/16	72/17	80/18
.9	0/9	9/10	18/11	27/12	36/13	45/14	54/15	63/16	72/17	81/18	90/19
10	0/10	10/11	20/12	30/13	40/14	50/15	60/16	70/17	80/18	90/19	100/20

a	p	q	b	
3	2	0	1	nuf
0	2	3	1	nuf
3	1	0	2	nuf
0	1	3	2	nuf
2	3	1	0	
1	3	2	0	
2	0	1	3	

(30 21)

aq > bp

.2 .5 .1 1,10

1 2 0 3

1 3 0 2.1

2 3 1 0

2 3 1 2

$a = .7$
$b = .2$
$p = .6$
$q = .1$

	No N_0		N_0		
a	1 2	a	1 6	a 7
b	2 1	b	6 1	b 6
p	6 7	p	2 7	p 2
q	7 8	q	7 2	q 1

$t = \dfrac{aq - bp}{(a+q) - (b+p)}$

$p = .5$ $\quad \alpha = \dfrac{.5 - .1}{.7 - .2} = \dfrac{.4}{.5}$ $\quad t = \dfrac{.7(.1) - .5(.2)}{.5 - .4} = \dfrac{.07 - .1}{.1}$ neg

$a = .8$ $\quad \alpha = \dfrac{6 - 1}{8 - 2} = \dfrac{5}{6}$ $\quad t = \dfrac{.8(.1) - .6(.2)}{} = .08 - .12$ neg

$b = .1$ $\quad \alpha = \dfrac{6 - 1}{7 - 1} = \dfrac{5}{6}$ $\quad t = \dfrac{.7(.1) - .6(.1)}{.6 - .5} = \dfrac{.07 - .06}{.1} = \dfrac{.01}{.1}$

$b = 1,5$ $\quad \alpha = \dfrac{6 - 1}{7 - 1,5} = \dfrac{5}{5,5}$ $\quad t = \dfrac{.7(.1) - .6(.15)}{}$ $\quad \dfrac{.07 - .09}{}$ neg

$.45$

$a = 6$
$b = 1$ $\quad p > a > q > b$
$p = 7$ $\quad 7 \; 6 \; 3 \; 1$
$q = 2$ $\quad 7 \; 6 \; 4 \; 1$

$q = 3$ $\quad \alpha = \dfrac{.4}{5}$ $\quad t = \dfrac{\overset{.18}{6.3} - \overset{.07}{7.1}}{(6+3) - (7+1)} = \dfrac{.11}{.1}$

$q = 4$ $\quad \alpha = \dfrac{.3}{5}$ $\quad t = \dfrac{6.4 - 7.1}{(6+4) - (7+1)} = \dfrac{.24 - .07}{1.10 - .8} = \dfrac{.17}{.20}$

$7.3 - 6.1$

$t \leq 1$

$\cancel{(a+q)}$

$(a-b) - (p-q)$

UNIVERSITA' DEGLI STUDI DI PADOVA

Valessi

29 nascondendo il sin. a sinistra
c'è trasparenza

29a non molto chiaro
nascond. a sinistra si può vedere
trasp.

29b si vede un po'

30 sì

31 sì

32 no

33 sì lateralmente bello

34 sì bello

35 no

36 tutti e due, a destra migliore

37 effetto strano. alternanza
quadrati trasparenti che
stanno metà in una striscia bianca
e metà in una nera

38 tutti trasparenti. bello

si quando c'è meno contrasto fra i chiari ← 39 no (come 37) (si vede tr. se è in ombra)

40 sì

41 trasp. bello
41a no: al più trasp. parziale nel vero
42 i laterali trasparenti; belli
43 trasp.
44 trasp.
45 si trasp.
46 parziale
48 a quadrati

Caporpi

Fig. 29A vede trasparente
tutta la parte centrale

38 più ombra che
trasparenza

39 laterali trasparenti
quello di mezzo è come
una macchia di sole

40 laterali trasparenti

41 trasp.

41a no

43 no

48 no

Anwendung der Durchschnittlichkeitsgleichung auf die Bereiche B und Q

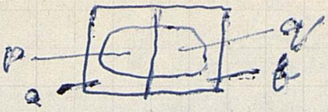
$$q = \alpha b + (1-\alpha) t$$

$$\alpha = \frac{q-t}{b-t}$$

C. $\boxed{b > q > t}$

D. $\boxed{t > q > b}$

AC	AD	BC	BD
$a > p > t$	$a > p > t$	$t > p > a$	$t > p > a$
$b > q > t$	$t > q > b$	$b > q > t$	$t > q > b$
		(Wiederholt AD ~~AD)~~ mit $b > a$)	



Durchsichtigkeitsgleichung $p = \alpha a + (1-\alpha) t$

$\alpha = \frac{p-t}{a-t}$

p = Reduzfarbe (Reduktionsfarbe)
t = Farbe der durchsichtigen phänomenalen Fläche
a = Farbe der durchgesehenen phänomenalen Fläche
α = Durchsichtigkeitsindex

Gültigkeitsbereich: $\cancel{1 > \alpha > 0}$
$0 < \alpha < 1$

1. $\frac{p-t}{a-t} > 0$

A_1	B_1
$(p-t) > 0$	$(p-t) < 0$
$(a-t) > 0$	$(a-t) < 0$
$\boxed{(p>t) \Longleftrightarrow (a>t)}$	$\boxed{(t>p) \Longleftrightarrow (t>a)}$

2. $\frac{p-t}{a-t} < 1$

A_2	B_2
$(p-t)^2 > 0$	$(p-t) < 0$
$\cancel{\frac{p-t}{a-t}(a-t)} < 1(a-t)$	$\cancel{\frac{p-t}{a-t}(a-t)} > 1(a-t)$
$(p-t) < (a-t)$	$(p-t) > (a-t)$
$\boxed{p < a}$	$\boxed{p > a}$

A_1	A_2		A_1	A_2
$p > t$	$a > p$		$t > p$	$p > a$
$a > t$			$t > a$	

A $\boxed{a > p > t}$

B $\boxed{t > p > a}$